KAMPALA INTERNATIONAL UNIVERSITY

COLLEGE OF EDUCATION, OPEN AND DISTANCE LEARNING

DEPARTMENT OF ACCOUNTING AND FINANCE

BACHELOR OF COMMERCE

QUANTITATIVE METHODS
COM1301

By

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INTRODUCTION

The Business School/CODL's aim is to become a market leader in the cost-effective provision of quality business management education leading to awards of certificates, diplomas and degrees internationally recognized and professionally acceptable. Throughout its history the business school/CODL has run a series of programmes primarily focused on business and management, which has been consistently, reviewed every three to five years to meet the contemporary needs of the market. The latest review was carried out in 2011 when the current curriculum was launched for six programmes: Business Administration, Bachelor of Commerce,

Businesses in the 21st century are experiencing profound challenges, which include the need to seek new market opportunities, develop new products that meet the changing demands of customers globally. The rapid growth of businesses and increasing transformations in the global economy has led not only to an increasing demand for specialists in the various management fields, but also to the need of a caliber of managers who are able to constantly adjust and innovate in the increasingly complex and volatile international business environment. It is upon this background that a modular system of teaching has been adopted to cope-up with the competitive environment of service delivery highly emphasizing on the concept of value for money.

The module enables a student to appreciate the concepts and examines the functions and roles of business in an organization. It presents a general overview and analysis of the main principles as a foundation for the more crystallized detailed description of policies, processes and practices, for purposes of setting ground for grooming the students in preparation for the challenging and dynamic field at the end of the course. For instance, accounting options seeks to provide answers to the need of management to maintain high and professional levels of competence in tracking, managing the inflows and outflows of resources in this volatile environment. It answers to the scientific managerial need for ensuring effective, efficient and productive use of resources and the ethical need for accountability and transparency.

The module explores various functional areas with accounting and finance, marketing and human resource management and examines in detail both conceptual and methodological tools that managers use to inform their decision making. Emphasis is placed on engaging with real life examples and applying course materials to specific familiar phenomena such as case studies. The main aims are to help students to understand the dynamics of today's business environment in the digital age.

The main aim of the study is to provide the student with an appreciation of the quantitative methods and its role in the managerial decision making process in the business environment.

The first unit introduces you to quantitative methods. These are the powerful tools through which production can be augmented, profits maximized, and costs minimized. Production methods can be oriented for the accomplishment of certain predetermined objectives. The study of quantitative techniques is a relatively new discipline, which has wide range of applications in the various fields like commerce, agriculture, industry, and marketing of late there has been a growing tendency for businesses to return to quantitative methods as a means of solving many of the problems that arise in a business or industrial enterprise. A large number of business problems in the relatively recent past have been given a quantitative representation with a considerable degree of success.

The second unit concerns the theory surrounding decision analysis. Decision analysis is the systematic approach of decision-making when faced with alternative courses of actions containing some degree of uncertainty or risk. Alternatives are choices available to the decision maker. Normally the results of a decision are not always known until some time after it has been made. In many cases, as we shall see, the result depends partly on chance event called state of nature, which are beyond the control of the decision maker.

In the third unit we will examines the meaning of queuing theory. Queuing theory deals with all aspect of a situation in which customers must wait for a given service. Waiting lines (queues) are part of everyone daily life examples of waiting lines are customers wait for a service, jobs wait to be processed, vehicles waiting at the traffic lights, planes wait to land.

The fourth unit explores the meaning of inventory models . Inventory refers to goods that are in stock. These are goods waiting to be sold on the market or to be used in production of other goods, the goods that are used to manufacturer other products are called raw materials - While those waiting to be sold are called finished products.

The fifth unit will introduce you to linear programming. Linear Programming is by definition a mathematical programming technique used in balancing the variable factors so as to achieve a pre-determined objective. For example, the major wish of every firm is to balance production capacity so as to minimize costs. In order to apply linear programming there must be a linear relationship between the factors some of which are variables and others non-variables.

The sixth, and final, unit introduces you to matrices and applications. Work carefully through the examples provided so that you understand the basic principles.

The majority of the units do not contain revision questions because of the examples cited in the text. You are expected to work through them. We hope you enjoy this study guide and will give it your best.

Unit 1
Introduction of quantitative methods

1.1 Meaning of Quantitative Techniques / Methods

Quantitative techniques are those statistical and operations research or programming techniques, which help in the decision-making process especially concerning business and industry. These techniques involve the introduction of the element of quantities that is they involve the use of numbers, symbols and other mathematical expressions. These quantitative methods are essentially helpful because they supplement judgment and intuition. These techniques evaluate planning factors and alternatives as and when they arise rather than prescribe causes off action. As such QM/Ts may be defined as those techniques, which provide the decision maker with a systematic and powerful means of analysis and help, based on quantitative data in exploring policies for achieving pre-determined goals. These techniques are particularly relevant to problems of complex business enterprise.

Numerous QM/Ts are available in modern times and they can broadly be put under two groups: -

1. Statistical techniques.
2. Programming techniques.

1.2 Programming / Operation Research Techniques

In our study for QM we shall concentrate on the programming techniques. Programming techniques are the so-called sophisticated quantitative techniques commonly known as **operations research techniques** (O R). The Management experts while talking about the QM in modern times have primarily in view the O.R techniques.

Examples of Important Operations Research Techniques.
Some of the important operations research techniques used in a business and industries today are:

Linear programming. This technique is used in finding a solution for optimizing a given objective such as profit maximization or cost minimization under certain constraints (problems). This technique is primarily concerned with the optimal allocation of limited resources for optimizing given functions. The name linear programming is used because of the fact that the model in such cases consists of linear equations indicating linear relationships between the different variables of the system. Linear programming techniques solve product-mix, distribution problems, investment, production scheduling of biz & industry. It is a technique used to allocate scarce resources in an optimum form in problems of production mix, distribution, and investments.

Key factors under this technique include an objective function, choice among several alternatives, constraints and the variables.

Waiting line or queuing theory. This deals with mathematical study of queues (lines) – The queues are formed when the current demand for service exceeds the current capacity to provide such service. Queuing theory concerns itself with a random arrival of customers where the facility is limited. Providing too much of capacity will mean idle time for servers and will lead to waste of money. On the other hand if the queue becomes long, there would be a cost in view of waiting the units in the queue. Queuing theory therefore aims at minimizing the overall cost due to servicing and waiting. In other words, the technique is used to analyze the visibility of adding facilities and to assess the amount and cost of waiting time. With its help we (you) can find the optimal capacity to be installed which will lead to a sort of economic balance between cost of service and cost of waiting.

Inventory control / planning. It aims at optimizing inventory levels. Inventory may be defined as a useful but idle resource, which has economic value for example raw materials, work in progress and finished goods. Inventory planning in fact answers two questions that is; "how much to buy and when to buy". Under this technique the main emphasis is on minimizing cost associated with holding of inventories, procurement of inventories and shortage of inventories.

Among the other aspects of QM we will be looking at, are:
- Breakeven analysis
- Game theory
- Decision theory
- Net work analysis

1.3 Role of Quantitative Techniques in Business and Industry

Quantitative techniques especially operations research techniques have gained importance since World War II in the technology of business administration.

These techniques greatly help in tackling the intricate and complex problems of modern business and industry. Quantitative techniques for decision-making are in fact examples of the use of scientific methods of management. Their role can be well understood under the following:

1. **They provide a tool for scientific analysis**. These techniques provide the executives with a more precise description of the cause and effect relationship and risk underlying the business operations in measurable terms and this eliminates the conventional intuitive and subjecting best approach in decision making which managements used to formulate their decisions decades ago. In fact, these techniques replace the intuitive and subjective approach of decision making by an analytical and objective approach. The use of these techniques has transformed the conventional techniques of operations and investment problems in business and industry. Quantitative mathematics / techniques thus encourage and enforce disciplined thinking about organisational problems.

2. **They provide solution for various business problems**. These quantitative methods are being used in a field of production, procurement, marketing, finance and others. Problems like how best can the managers and executives allocate the available resources to various products so that in a given time , the profits are maximized or the cost is minimized. Is it possible for a n industrial enterprise to arrange the time and quantity of orders of its inventory such that the overall profit with given resources is maximum.

3. **They enable proper deployment of resources**. Q.Ts render variable help in proper deployment of resources e.g. programme Evaluation and Review Technique (PERT) enables us to determine the earliest and latest times for each of the events and activities and there by helps in identification of the critical path. All these help in the deployment of resources from one activity to another. This technique thus helps to determine the time required for completing an event or project its self by a specified date.

4. **They help in minimizing, waiting and servicing cost**. The waiting line/queuing system helps the management in minimizing the total waiting and servicing cost. It also analyses the visibility of adding facilities and thereby helps the business people to take a correct and profitable decision. Example is the number of servers on an ATM in a banking system

5. **Enable the management to decide when to buy and how much to buy**. The main objective of the inventory planning is to achieve balance between the cost of holding inventory and the benefits enjoyed from holding it. Hence the technique of inventory planning enables the government to decide on when to buy and how much to buy.

6. **They render great help in optimum allocation of resource**. Linear programming technique is used to allocate scarce resources in an optimum form. Cases in point include problems of product scheduling, product mix. Modern managers in resource allocation and in effecting optimal assignments thus popularly use linear programming.

7. **Quantitative techniques facilitate the process of decision-making**. Decision theory enables the business manager to select the best course of action when information is given in mobilization/scattered form. In this case, a problem is identified, alternative courses of action are formulated, best alternatives courses of action are selected and finally implemented

1.4   Quantitative Techniques and Business Management

Quantitative methods render valuable services in the field of business management. They for example help directive authority/management in optimum allocation of various limited resources. For example machines, money, materials etc can be allocated to different competing opportunities on an objective basis for achieving effectively the goals of a business unit. They help a CEO in broadening management vision and perspectives in the choice of alternative strategies to decision problems such as forecasting man power, production capacities and capital requirements and plans for their acquisitions.

1. **Q.M are useful to a production management in:**
   - Selecting the building site for a plant, scheduling and controlling its development and designing layout.

- Locating within a plant and controlling the movements of the required production materials and finished goods inventories.
- Scheduling and sequencing production by adequate preventing maintenance with optimum number of operatives by proper allocation of machines.
- Calculating an optimum product needs.

2. Quantitative techniques (QT) are also useful to the personnel management to find out;

- Optimum manpower planning/requirements. (E.g. through Linear Programming and Network Analysis).
- The number of personnel to be maintained on the permanent or fulltime basis
- The number of persons to be kept in a work pool such as secretaries.
- The optimum form of sequencing and recruiting of personnel to a variety of jobs. E.g. Accountants, cashiers etc.
- In studying personnel recruitment procedures and labour turn over cases.

3. Q.T equally help the marketing management: (LP & inventory planning)

- To determine where distribution points and ware housing facilities should be located, their size, quantity to be stocked and the choice of customers.
- The optimum allocation of sales budget to direct selling and promotional expenses.
- The choice of different media of advertising and advertising strategies.
- The consumer preferences relating to size, colour, packaging e.t.c from various products as well as to out bid and out unit competitors.

4. Q.T are also very useful to financial management especially in finding: (investment analysis)

- Long range/term capital requirements as well as how to generate these capital requirements.
- Determining optimum replacement policies.
- Working out a profit plan for a firm.
- Developing Capital investment plans
- Estimating credit and investment risks

In addition to all these, Q.Ts provides the business executives with an understanding of the business operations, which gives one new insights and capability to determine better solutions for several decision-making problems with great speed, competence and confidence. When applied on the level of management where policies are formulated, the Q.Ts are seen in the executive in an advisory capacity but on the operation level where production, personnel, purchasing, inventory and administrative decisions are made, they provide management with the means of handling and processing information.

1.5   Limitations of Quantitative Techniques

Q.M though is of great aid to management, they cannot be a substitute of decision making. The choice of the criterion (way out) such as what is actually the best alternative for business enterprise is still that of an executive who has to fall back upon his experience and judgment. This is so because of the several limitations of Q.Ts.

1. **The inherent limitation concerning mathematical expression**, Q.Ts involve the use of mathematical models, equations and similar other mathematical equations. Assumptions are always incorporated in the derivation of equations or models, such an equation may be corrected for the solution of the business problems when the underlying assumptions and variables are present in a concerning problem. If this question is not tackled with care, then there always remains the possibility of wrong application of the Q.T.

2. **High costs are involved in the use of Q.Ts**. Q.Ts usually prove to be very expensive services of specialized persons and are invariably called for while using Q.Ms. As such, only big concerns (organisations) can be pursuing such techniques. Even in big organisations, we expect that Q.Ts will continue to be of limited use, simply because they are not in many cases worth their cost. As opposed to this, the technical manager exercising intuition and judgment may be able to make a judgment very inexpensively. Thus the use of Q.Ms is a costly affairs and this in fact constitutes a big and important limitation of such method.

3. **They don't take into consideration the intangible factor i.e non-measurable human factors**. Q.Ms make no allowance for intangible factors such as skills, attitude, and vigour of the management in taking decisions, but in many instances, success or failure hinges upon the consideration of such non-measurable factors. There cannot be

any magic formula for getting an answer to management problems, many depends upon proper managerial attitudes and policies.

4. **Q.Ts are just the tools of analysis and not the complete decision making process.** It should always be kept in mind that Q.Ts, whatsoever, it may be, alone cannot make a final decision. They are just tools and simply suggest best alternatives, but in the final analysis many business decisions will involve human elements. Thus Q.M is at best a supplement to rather than a substitute for management; subjective judgment is likely to remain a principle approach to decision making.

Conclusion

The above description of the meaning and use of various Q.Ts nearly explains the role of such techniques in the field of business and industry. Because of their usefulness the study of Q.Ms has gained increasing importance especially in the modern times. There is growing awareness amongst the people that unless they make themselves familiar with Q.Ts, they will not be able to understand and appreciate the problems of modern business units. With computer facilities becoming wide spread, the significance of Q.Ts is likely to grow in the coming year's inspite of the above stated limitations. Not only that many new Q.Ts may evolve to solve the ever increasing problems of the business and industrial world. However, these techniques should be taken only as an effective aid to the decision-making process, they can never be a substitute for human skills, experience and judgment.

Review Questions

1. Q.Ts is a compliment and not a replacement of experience, skills and knowledge of an executive in decision making discuss the concept in relation to the capabilities and limitations of Q.Ts. Narrate fully the role of Q.Ts in the field of business and industry; give examples in support of your answer. Do you think the day will come when all the decisions in business units are made with the assistance of Q.Ms? Give reasons to your answer.
2. "Q.Ms has considerable accomplishments to its credit, in light of this therefore appraise the fact that all decisions in business units are made with the help of quantitative methods.

Unit 2
Linear Programming
2.1   Introduction

Linear programming may also be defined as the mathematical modeling technique designed to optimize the usage of limited resources. For example we may want to find the maximum possible sales or the minimum operating cost subject to certain conditions (constraints) such as raw materials, manpower, production capacities etc.

The company's cost and profits depend on the availability and use of the various resources or components such as manpower (labor), raw materials. The cost of these components, the profit of each product and the demand of the product can determine the success of the organization.

If the relationships among the various resources, production requirements, the costs and profits are all linear, then these activities may be planned (programmed) in the best possible (optional) way by means of linear programming (LP).

Fortunately, many relationships can be linearised i.e reduced to the kind of linear form. Programming is used to refer to mathematical technique used to obtain the best possible solution to the problem involving ltd resources.

Linear programming. This technique is used in finding a solution for optimizing a given objective such as profit maximization or cost minimization under certain constraints (problems). This technique is primarily concerned with the optimal allocation of limited resources for optimizing given functions. The name linear programming is used because of the fact that the model in such cases consists of linear equations indicating linear relationships between the different variables of the system. Linear programming techniques solve product-mix, distribution problems, investment, production scheduling of biz & industry. It is a technique used to allocate scarce resources in an optimum form in problems of production mix, distribution, and investments.

2.2   Application

- Job assignment / allocation (maximizing profits and minimizing costs, how many people are supposed to do a certain job.
- Production scheduling: How many employees will come during the day and during the night (night shifts and day shifts).
- Transportation problem. We don't want people to collide e.g. cocacola trucks must take different routes to supply the soft drink and meet peoples' concerns about quantity demanded.
- Purchasing problem. How much can we buy at the same time so that I maximize the profits and minimize any costs?
- Product mix e.g. good components e.g. CocaCola, Bell and tusker
- Investment problem – where best can one invest e.g. in Uganda or Kenya, Rwanda or can I go in trade, University investment e.t.c
- Capacity problems how much can you supply in market or how much can I keep as stock.
- Location problem i.e where can I locate e.g. a university

Requirements for Linear Programming
- Objective function i.e it is concerned about the target or ambition. The objective should either maximizing profits or minimizing costs
- Constraints i.e objective function should be subject to constraints.
- Mathematical relationship. Objective functions and constraints should be able to be expressed on a mathematical relationship
- Non- negatively condition. We should not have negative attitudes in our companies.
- Feasible solution i.e you must have an area for all possible solutions for the problems.
- Optimum solution that is best alternatives (choices), hence maximizing profits or minimizing costs
- Unfeasible solution i.e it should not be considered because it is unwanted.

2.3   Basic Assumptions for Linear Programming

Certainty. We assume that conditions of certainty exist i.e numbers in the objective function and constraints are known with certainty and do not change during the period under study for example you are certain about the existence of KIU, MOI University because of changes in fees, you must look for a cheaper university. It is all about changes in an environment.

Proportionality. We assume proportionality exists in the objective function and constraints. This implies that if production of one unit of a product uses 3 hours of a particular scarce resource, then making 10 units of the same product uses 30 hours of the resource

For example if;
1 product requires 3 hrs
10 products = 3x10 = 30 hrs

Non-negativity. We assume that all answers of variables are non-negative. Negative values of physical quantities are impossible situations. You simply can not produce a negative number of chairs, cars etc

2.4   Graphical analysis of problems
Step 1: To determine what your objective function is:
Step 2: To assign variables (decision variables) to each unknown quantity
Step 3: Express of the objective as a function of the decision variables.
Step 4: Express the constraints or the limited resources as inequalities or equalities in terms of the variables.
Step 5: include any additional implicit constraints
Example 1

Sadolin Company produces two types of paint that cream and Maroon. The two products go through crushing and mixing processes. Crushing has 60 hours available and mixing 48 hrs available. Manufacturing each carton of cream requires 4 hrs in crushing and 2 hours in mixing, whereas maroon requires 2 hours in crushing and 4 hours in mixing. If the profit is $8 per tin of cream and $6 per tin of maroon.

Required:
(a)   Determine the best possible combination of tins that should be produced for sale in order for Sadolin to maximize her profits approach.
(b)   What is the maximum profit when both cream and maroon paints are produce?
(c)   Discuss any 5 applications of LP in the business world.

Solution

Tabulation of the information

Processes Types of paint	Crushing (X)	Mixing (Y)	Profits
Cream (C)	4 hours	2 hours	$ 8

Maroon (M)	2 hours	4 hours	$ 6
TOTALS	60 hours	48 hours	Max-Z

Let:
Cream be represented by C
Maroon by M,
Crushing be represented by X
Mixing by Y
Over all objective function (Max-Z) = 8C + 6 M.
Assume you are to produce C quantities of cream at a profit $8 per tin, the maximum profit per tin will be 8 X C = $8 C. Similarly the maximum profit for maroon will be 6M, if you are to produce M quantities of maroon.
Therefore the overall function will be Max-Z = 8 C + 6 M.

STEP II
Form the constraints because the objective function is subject to constraints
For example constraints
Crushing: $4C + \underline{2M} < 60$ Hrs……………….C_1 (It should be less than 60)
Mixing: $2C + 4\underline{M} < 48$ Hrs………………… C_2
Important sign should be < for maximizing profits.
Important sign should > for minimizing costs
$C > 0$………………………..C_3 non negatively
$M > 0$……………………….. C_4

Step III
Transform the constraints into linear equations
That is $4C + 2M = 60$……………………. C_1
$2C + 4M = 48$………………………….… C_2
$C = 0$ ……………………………. C_3
$M = 0$ ……………………………. C_4

Step VI
Using the linear equations, we can compute the coordinates of the lines to plot on the grid reference.
E.g. $4C + 2M = 60$:

Using the basic rule, if,

C	0	15
M	30	0

Coordinates then become: (0, 30) and (15, 0)

2C + 4M = 48

C	0	24
M	12	0

Step V
Plot the linear equations on scale for X and Y axes (Grid reference)
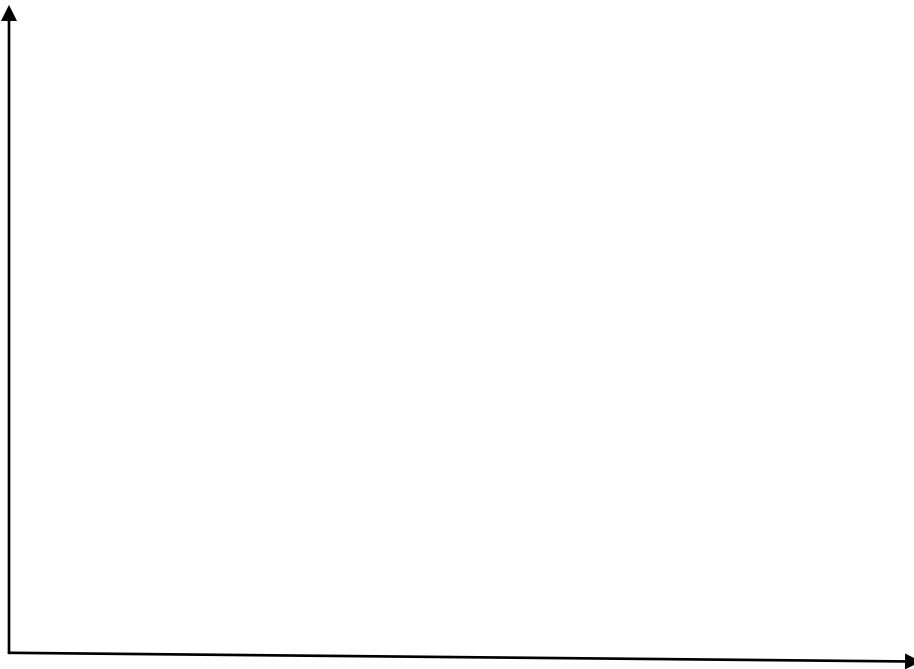
Step VI
Get coordinates of the feasible region

A:
B:
C:
D:
STEP VII
Solve a simultaneous equation to get the coordinates for the point, which is not cutting the axis. (Substitution or elimination or matrix methods)

Step VIII
Using the objective function, we can multiply out to get the possible combination using the points obtained from the feasible region of the grid reference.

Example Two
Avis Company limited produces two types of cream that is herbal and lemon, the production of each type of cream involves 3 processes that is processing, assembling and finishing. The production of herbal cream involves 2 hours of processing, 3 hours of assembling and 1 hour of finishing. Lemon cream requires 1 hour of processing, 6 hours of assembling and 1 hour of finishing. Total time available is 160 hours for processing. 480 hours for assembling and 100 hours for finishing. The profit per container of herbal cream is 100/= and 8/= for Lemon cream respectively.

REQUIRED:
What combination maximizes profits when both creams are produced?
 What is the maximum profit when both creams are produced?

Processes / Types	Processing P	Assembling A	Finishing F	Profits
H	2 hours	3 hours	1 hour	100/=
L	1 hour	6 hours	1 hour	8/=
TOTAL	160 hours	480 hours	100 hours	Max z

Key
P   Processing
A   Assembling
F   Finishing
H   Herbal
L   Lemon

Max z Objective function

Objective function
$100H + 8L = Max z$

Constraints to the objective function.
$2H + L < 160$………………C_1

$3H + 6L < 480$...............C_2
$H + L < 100$...................C_3
$H > 0$..........................C_4
$L > 0$............................C_5

Equations derived from the constraints
$2H + L = 160$
$3H + 6L = 450$
$H + L = 100$
$H + 0$
$L + 0$

Finding co-ordinates of the lines to plot on the graph

Equation 1: 2H + L + 160

$2H + L = 160$

H	0	80
L	160	0

$2 \times 0 + L + 160$ H   L   H   L
$L = 160$ (0, 160)   (80, 0)

$2H + L + 160$
Equation II: 3H + 6L = 480

$2H + L \times 0 = 160$

H	0	160
L	80	0

$\underline{2H} = \underline{160}$ H   L   H   L
$2 \qquad 2$ (0, 80)   (160, 0)

$H = 80$

Equation II H + L = 100

H	0	100
L	100	O

                                H   L   H   L
                                (0, 100)   (100, 0)

Grid reference showing the linear equations.
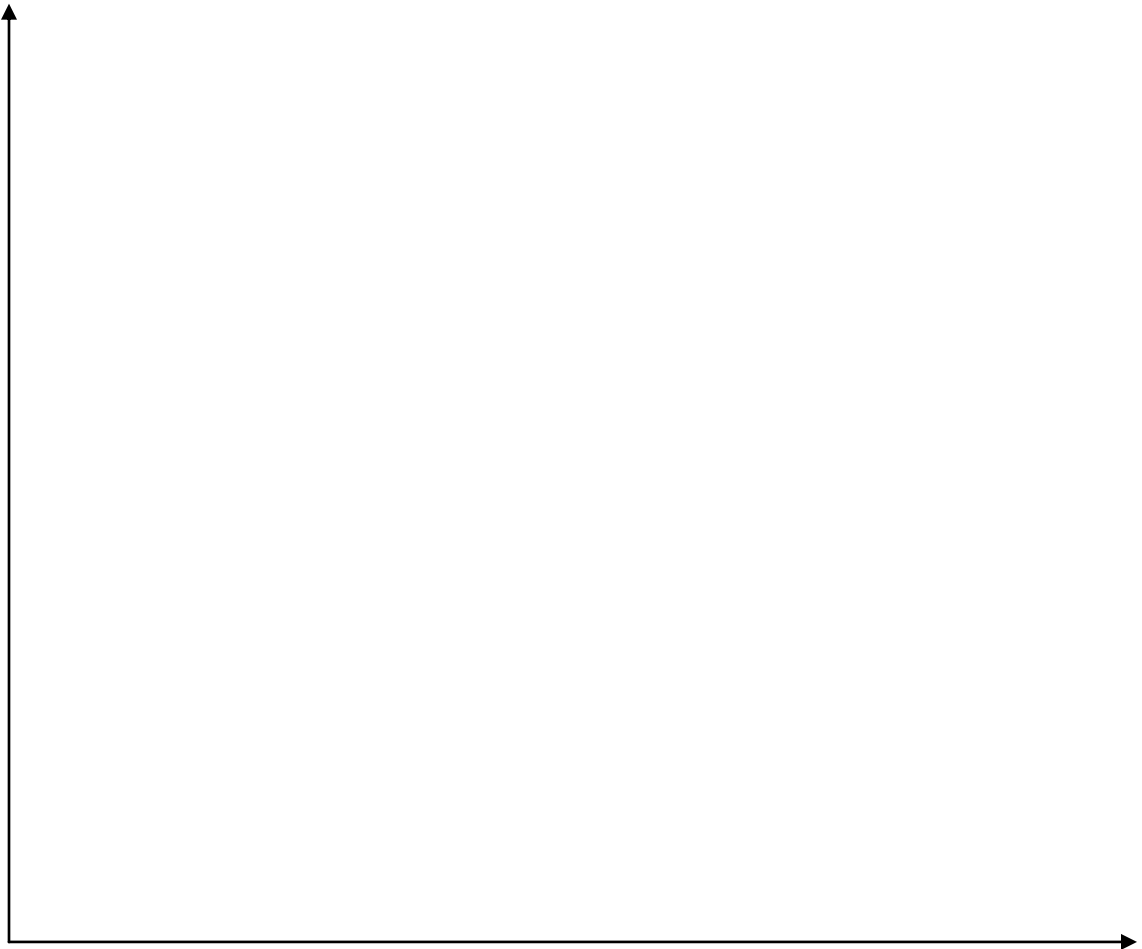
Coordinates of the points

A;   (0, 0)
B;   (0, 80)
C;   (40, 60)?
D;   (60, 40)?
E;   (80, 0)

Point C (Cuts equation (i) and equation (iii))
Solve simultaneous equations.

$2H + L = 160$……………………..Eqn. (i)
$H + L + 100$………………….…..Eqn. (iii)
By elimination method – subtracting
$2H + L = 160$

H + L = 1000
H + 0 = 60
H   = 60

Substitute in any of the two equations
60 + L = 100   e.g.   60 + L = 160
    L = 100 – 60   120 + L = 160
    L = 40   L = 160-120
    (40, 60)   L + 40
Point D (cuts equation (ii) and equation (iii))
Solve simultaneous equation using elimination method.
3H + 6L = 480
H + L = 100
By elimination
1 x (3H + 6L = 480)
3 x (H + L = 100)

 3H + 6L = 480
 3H + 3L + 300
 0 + 3L = 180
 3L =_180
 3   3
 L = 60

Substitute in any off the above equations:
H + 60 =100
H = 100 – 60
H = 40
(60, 40)

Apply the co-ordinates in the objective function

Using 1000H + 8L = Max Z
A (0, 0) = (100 X 0) + (8 X 0) = 0/=
B (0, 80) = (100 X 0) + (8 X 80) = 640/=
C (40, 60) = (100 X 40) + (8 X 60) = 4480/=
D (60, 40) = (100X 60) + (8 X 40) = 6320/=
E (80, 0) = (100 X 80) + (8 X 0) = 8000/=
a (i) The combination that maximizes profits if both creams are marketed is combination D where 60 units of herbal cream are marketed and 40 units of lemon cream are marketed at profit of 6320/=

ii)   Maximum profits when both creams are produced is 6320/=

b) Areas of application include;
- Production scheduling
- Transportation problem
- Purchasing problem
- Product mix
- Investment problem
- Capacity problem
- Location problem
- Job assignment/allocation

Example Three

Toyota Company produces two types of vehicles of Corolla and Hilux, which must be processed, assembled and finished. The processing machine has 48 hours, available, the assembling machine 30 hours, finishing 50 hours. Then production of the corolla requires 1 hour per unit on processing, 1 hour per unit on assembling and 2 hours per unit on finishing. Similarly each unit of Hilux requires 4 hours on processing, 2 hours assembling and 1 hour on finishing respectively. A unit profit is 5 dollars for a Corolla and 10 dollars for a Hilux

Required
Combinations of vehicles that will maximize Toyota company's profits

Processes Types	P	A	F	Profits
C	1 hour	1 hour	2 hours	$5
H	4 hours	2 hours	1 hour	$10
TOTALS	48 hours	30 hours	50 hrs	Max Z

Key
P   Processing
A   assembling
F   Finishing
∏   Profits
C   corolla
H   Hilux
Max z   Objective Function

Objective function

$5C + 10H = \text{Max } Z$

Constraints to the objective function

$1C + 4H < 48$ ....C_1

$C + 2H < 30$........................ C_2

$2C + H < 50$......................... C_3

$C > 0$............................... C_4

$\text{If} > 0$..................................C_5

Equations derived from the constraints

$C + 4H = 48$.............................equation (i)

$C + 2H = 30$.............................equation (ii)

$2C + H = 50$.............................equation (iii)

$C = 0$......................................equation (iv)

$H = 0$.......................................equation (v)

Equation (i)

C	0	48
H	12	0

C	H	C	H
(0, 12)		(48, 0)	

Equation (ii)

C	0	30
H	15	0

C	H	C	H
(0, 50)		(25, 0)	

Equation (iii)

C	0	25
H	50	0

(0, 50)   (25, 0)

$2C + H = 50$

C	0	25
H	50	0

```
       H   C
A:     (0, 0)
D:     (0, 25)
R:     (3.33, 13.35)
I:     (9, 12)
N:     (12, 0)
```

Solve simultaneous equation
Equation (ii) & (iii)

$$C + 2H = 30$$
$$2C + H = 50$$

By elimination   Substitute in any equation
2 X (C +2H = 30)   2C + 3.33 = 30
1 X (2C + H = 50)   2C = 30-3.33
2C + 4H = 60 $\underline{\quad\quad 2C = 26.7}$
$\overline{2C + H = 50}$ 2   2
$\overline{0 + 3H = 10}$ C = 13.35
$\quad \underline{3H} = \underline{\ 10\ }$ 2 X 13.35 + 3.33 = 30
$\quad\quad 3 \quad\quad 3$ 26.7 + 3.3 = 30
$\quad\quad$ H = 3.33

Equation I and equation (ii) point I
 C + 4H = 48   C + 2 X 9 = 30
C + 2H = 30   C + 18 = 30
0 + 2H = 18   C = 30 - 18
H = 9   C = 12

5C + 10H = Max Z Substitute in the objective function
Required

A; (0, 0) = (0X5) + (0X10) = 0
D; (0, 25) = (25X5) + (0X10) =125
R; (3.33, 13.35) = (13.35X5) + (3.33X10) = 100.05
I; (9, 12) = (12X5) + (9X10) = 140
N, (12, 10) + (12X10) = 120.

Combination is I where 12 units of Hilux are marketed and 9 units of corolla are marketed.

Example Four
Dunlop Company manufactures two types of motor vehicle tyres model P and model R. Model P sells for US dollars 95 per tyre and costs US $ 85 per tyre to make whereas model R sells for US $50 per tyre and costs US $ 42 to make. To make the model P type it requires two hours in machine A and four

hours in machine B. On the other hand to make one model R tyre it takes nine hours on machine A and 3 hours on machine B. Production scheduling indicates that in a week machine A 13 available for at most 36 hours and machine B for at most 42 hours.

Determine

a) How many of each tyre should a company make in a week in order to maximize the profit
b) What is the maximum profit?

Example Five

Shell operates two refines with different production capacities, refinery A can produce 4 million litres of petrol, 2 million litres of diesel and 1 million litres of Kerosen per day. On the other hand refinery B can produce 1 million litres of petrol, 3 million litres of diesel and 4 million, litres of Kerosene per day. The company has made a contract with a dealer company to provide 24 million litres of petrol, 42 million litres of diesel and 36 million litres of Kerosene. The cost of running refinery A is US $ 1,500 per day and refinery B is US $ 2400 per day.

Determine

(a) The minimum cost of the two refineries
(b) The quantities of fuel produced by the refineries of the minimum cost

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Review Questions

1. Explain the basic assumptions of linear programming
2. Discuss the applications of linear programming

Unit 3
Queuing Models
3.1   Introduction
Queuing theory deals with all aspect of a situation in which customers must wait for a given service. Waiting lines (queues) are part of everyone daily life examples of waiting lines are customers wait for a service, jobs wait to be processed, vehicles waiting at the traffic lights, planes wait to land.

3.2   Purpose of Queuing
   Waiting lines (queues) form when the current demand for a given service exceeds the capacity to provide the service. In most cases additional service facilities could be provided to decrease queue or to prevent queues from forming; however the cost to provide the additional service may cause the margin of profit to drop below unacceptable level. On the other hand excessively long waiting line result in loss of sales and loss of customers' goodwill so there need to balance the cost associated with waiting against the cost associated with prevention of waiting in order to maximize the profit or minimize cost.

   A queuing system involves customers arriving at the service facility (server). Customers are generated from a source; this source can be finite or infinite. A finite source e.g. ATM machines for a given bank that require to be serviced, alternatively infinite source is forever abundant e.g calls arriving at a telephone exchange.

   On arrival at a facility customers can be serviced immediately or wait in queue, when the service is occupied when the facility completes the service it automatically pulls a waiting customer if any, from the queue. If the queue is empty, the facility becomes idle until the new customer arrives.

3.3   Core Elements of Queuing Systems

There are so many types of queuing systems but all can be classified according to the following characteristics:

The input or arrival process.
This includes the distribution of the number of arrivals per unit of time, the

number of queues that are permitted to form, the maximum queue length

and the number of customers desiring the service.

The service process

This includes the distribution of the time to service the customers the number of servers (facilities) and the arrangement of servers (the arrangement may be in parallel or in series).

The queue discipline

This represents the manner or order in which customers are selected from the queue.

- e.g first come, first serve (FCFS) or in finance, first in first out (FIFO)
- Last come first served (LCFS) or last in first out (LIFO)
- Service in random order or (SIRO)
- Priority selection such as emergency services in hospitals, rush jobs are processed ahead of regular jobs. Customers may jockey from one queue to another in hopes of reducing their waiting. They may also balk from joining the queue altogether because they anticipate the long delay or they may revenge from the queue because they have waited for so long.

Examples

1. Single Queue Single Server
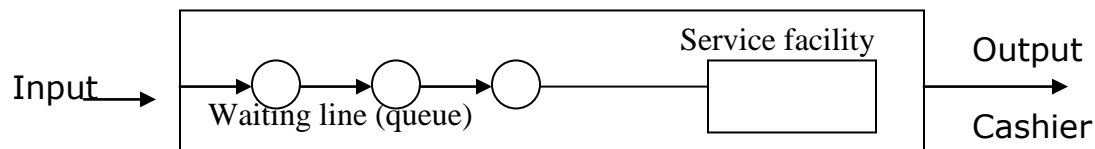
e.g Grocery

2. Single queue multiple servers in parallel
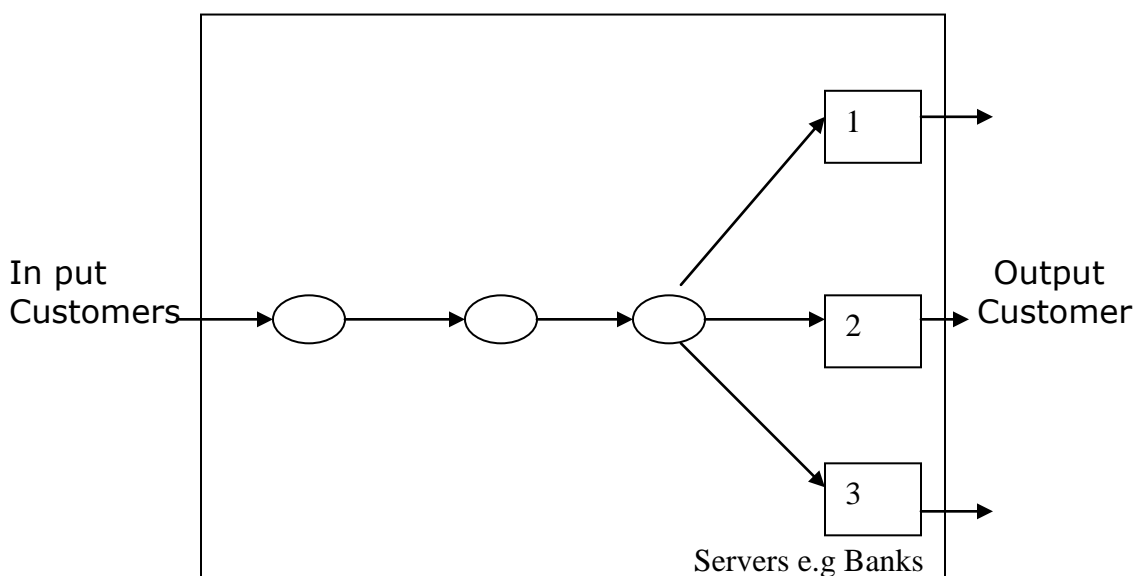
3. Single, queue, multiple servers, in series
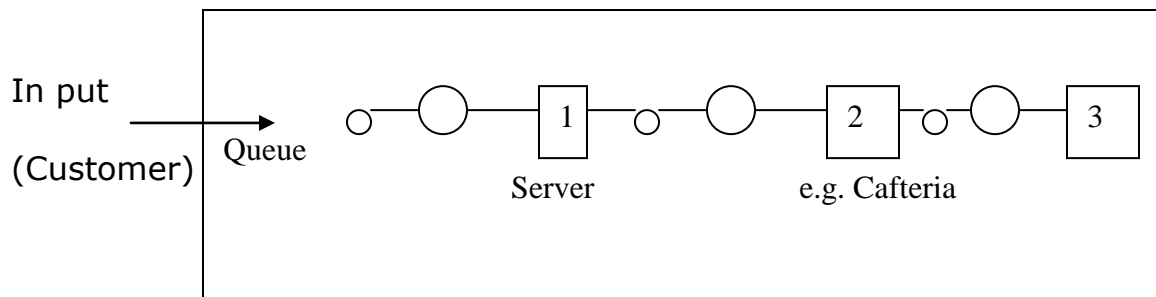
4. Multiple queue, multiple servers

- Output (customer)

- Supermarkets

- Fueling station
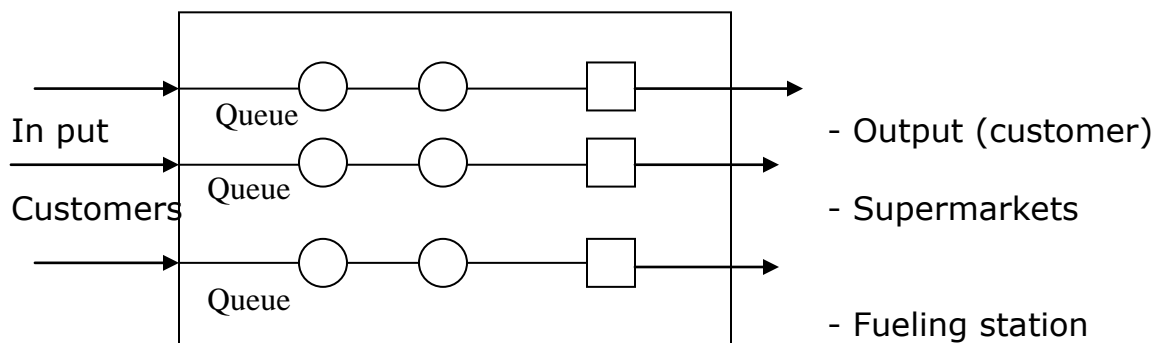
3.4 Assumptions in the queuing theory
- The service is provided on a FIFO basis
- Customers arrive complete at random but at a given average rate
- The queuing system is in a steady state condition.

3.5 Mathematical calculations

A number of arrivals at a facility per unit of time has a poisson distribution with mean (m) and the rate of arrival (Y). The time between consecutive arrivals (inter arrival time) has an experimental distribution with mean (X).

Poisson Probability distribution

$$P(x) = \frac{e^{-\lambda} \lambda^x}{X!} \qquad X = 0, 1 \ldots\ldots\ldots\ldots$$

Exponential Probability distribution
$$P(x) = \lambda e^{-\lambda x} : \quad x = 0, 1, 2\ldots\ldots\ldots$$

Suppose we have the following
λ = Average arrival rate of customers in the system
M= Average service rate of customers in the system.

We can derive the following:

L_s = Average number of customers in the system.
Lq = Average number of customers in the queue.
Ws = Average time a customer spends in the system.
Wq = Average time a customer spends in the queue

$$P_n = (\lambda/m \ (1-\lambda/m)$$

Pn = Probability of n customers in the system.
We shall assume that m > λ i.e the mean service rate is greater than the mean arrival rate otherwise the waiting line will become infinitely large.

(1) $\quad L_s = \dfrac{\lambda}{m - \lambda}$

(2) $\quad L_q = \dfrac{\lambda^2}{M(M-\lambda)}$

(3) $\quad W_s = \dfrac{I}{M - \lambda}$

(4) $\quad P_s = \left[\dfrac{\lambda}{M} \right]^2 \ (1 - \lambda/M)$

(a) $\quad L_s = \lambda W_s$

(b) $\quad L_q = Wq\lambda$

(c) $\quad L_s = L_q + \lambda/M$

Review Questions

1. Customers arrive at one person's salon according to a poison process with a mean inter-arrival time of twenty minutes. Customers spend an average of 15 minutes for hair cutting within a period of one hour, find

   (a) The expected number of customers at the Saloon.
   (b) How much time will a customer spend in the queue?
   (c) What is the probability that a customer will not have to wait for a haircut?

2. What is the probability of having more than 3 customers in the system?

Unit 4
Decision Making Theory
4.1   Introduction
Decision analysis is the systematic approach of decision-making when faced with alternative courses of actions containing some degree of uncertainty or risk. Alternatives are choices available to the decision maker. Normally the results of a decision are not always known until some time after it has been made. In many cases, as we shall see, the result depends partly on chance event called state of nature, which are beyond the control of the decision maker.

Managers spend a large amount of time making decisions. These decisions are valid and are made under a wide range of circumstances. The decisions involve different priorities and also differ in terms of routines and noverity. The decisions are required to solve problems that arise when an actual state of affairs differs from a desirable state. Further more in deriving these decisions so as to solve a problem several facets of the problem must be analyzed and several alternative options must be considered.

Decision making is thus particularly important in any managers job e.g. the manager should determine how   to re-invest profits, should which buy or lease a certain asset, what is the significance of the current job assignment Kisaki efficiency,   what are the current implication   of the budget. The decision taken in determining these answers should be necessarily important in their own. However, they hold a high level of significance in respect to the profitability of the business for which the manager is primarily responsible. In view of the scarcity of resources available to the manager, the decision he/she   makes   must be in s sense correct so that   he does not deplete those scarce resources and eventually finds himself out of the job.

Q.Ms in decision making thus taken on, adds significance if we recognize their importance in aiding the manager logical decisions. These techniques force the decision maker to make his assumptions explicit at every stage of reasoning by providing an orderly approach to the analysis at hand. Several advantages arise out of applied Q.Ts to an analysis:-

- They are concise and precise in terms of language
- There exists a wealth of mathematical expressions theory that can be applied   to the process of analysis.
- The use of Q.Ts force us to carryout structured argument (statements) by stating explicitly all the assumptions underlining our arguments.

On the basis of these advantages, the use of Q.M can be seen as a vehicle that transports us efficiently from postulates to conclusions in an efficient form, these methods are thus only means to an end and must thus be understood in principle if they are to aid the decision making process. Time must be taken to know the skills of applying the Q.T to the analysis of economic and business phenomenon.

4.2   The Planning Process

In the course of carrying out any economic activity, the prayers must apply in planning, planning is a fundamental   process of establishing objectives and suitable cause of action before taking a decision. It is the basis process used to select goals and determining how to achieve them. It is a pre-requisite for leading and controlling, planning process involved managers in a continule series of making decisions and the quality of the decisions they make reflect the effectiveness of the planning process.

Planning can be recognized as consisting of four stages that are adoptable to all organisational levels.

STAGE I   Goals Establishment
In order to avoid a thin resource spread and also identify the priorities, it is important to establish goals. Goals also form a basis for control where feedback can be obtained. The goals established are the start of the planning process help explain the objectives of the economic activity and motivate the players if they are realistically achievable.

Stage Ii: Situation Of Analysis
Having established the objectives that we w ant to achieve must carryout an analysis of the operating environment. This involves an examination of internal organisational factors and external environmental factors that (directly and indirectly) affect the achievement of the set goals. The current organisational status must be established by providing an answer to the question "Where are we", "where do we want to move", and "how shall we move there". This involves the process of drawing a situational map to find our location.

Stage Iii: Determination Of Aids And Barriers
The third stage of a planning process involves an analysis of the organisational strength and weaknesses and opportunities and threats (SWOT) analysis. Here we wish to establish the strong technical team comparative advantages enjoyed by the organisation for example we may have a strong a strong technical team and a high quality product

(strength). But employ a poor marketing strategy (weaknesses). Exogenously, opportunities may exist in the form of a good infrastructure and a big market. Threats may be in a form of substitutes and new market entrance (threats). The planning process should actually take these factors into consideration (opportunities).

Stage Iv: Development On Aset Of Action
When the above stages have been completed, the planner on the basis of analysis carried out should be able to obtain a wide perspective of the plans required to optimize his/her scarce resources and also achieve her or his objectives. The planner must then develop a set of actions (plans) in line of the processing analysis in such a form as will lead to the best return on the investment.

4.3   Tools and Techniques For Planning

The planning process as we have observed is multi-stage and multi dimensional exercise of the number of variables. The tools and techniques we apply this process should be able to take into account the multiple aspects of the problem at hand. There is also, as far as it is predictable, take into consideration the assumptions that underlie our plan. Two of the most tools of planning are forecasting and scheduling.

Forecasting: As a tool for planning involves an attempt to predict outcomes and project future trends. Forecasts approximate future trends closely so as to guide present actions in a valid and purpose full w ay. The usefulness of forecasts lies in the fact that the results derived from their use can be used as the basic assumption for decision making and planning. Forecasts a re broadly of two types.

a) Forecasts of internal invests that will be partly influence by the organization. For
    example financing and human resource.
b) Forecasts of basic economic, social and political, variables, external to the
    organization. E.g. taxation and macro investment policies.

Forecasts can also be either qualitative or quantitative. Qualitative forecasts are appropriate where 'hard' data is scarce or difficult to obtain e.g. when introducing a new product or technology. The judgments that we make (due to lack of precedent) involve the use of subjective judgment and expert opinions.

On the other hand, quantitative forecasts extrapolate from the past data that is sufficiently available to specify relationships between key variables.

Scheduling. It is the process of pre-determining process or actions and the periods during which these activities will occur. The greatest advantage of scheduling arises out of the control benefits, it affords a manager. Through it uses, the manger can determine where this is not happening take corrective action

4.4   Decision Making Process

Decision-making is a function of the management process .It may be defined as a process of   identifying   and selecting suitable cause of action to solve a specific problem. It involves two aspects.

1) Choice making. The process of choosing the best option from the set of viable alternative options. Choice making is narrow in the sense that its ltd to selecting an alternative which failed to be the best after more detailed analysis.
2) Problem solving, Finding and implementing the causes of action to correct an unsatisfactory situation. It involves recognition of the methodology that may be to solve problems.

4.5   Types & Stages of Decision Making

Managers are required to make decisions on a continuous basis, sometimes even without having the time of thinking about the implementations of those decisions. The decisions that managers make can be classified into two categories.

1. **Programmed Decisions**. These are made in accordance with their habit, rule and procedure. They are usually mechanical and repeated e.g. What to pay a newly hired   employee. Programmed decisions can also be made when solving complaint problems especially when these problems are anticipating and the procedure for solving them has been worked out.

2. **Non-programmed decisions**. These   deal   with   un   usual   or exceptional problems – such problems are not recurring and not covered by policy e.g. what to do with and failing   product line.

Stages in Decision Making Process
There are five stages in the decision making process.

```
┌─────────────────────────────────┐
│ Recognize a need for a decision │
│ (Define the problem)            │
└─────────────────────────────────┘
        ↑    ↓
┌─────────────────────────────────────────┐
│ Formulate objectives  and identify      │
│ alternatives                            │
└─────────────────────────────────────────┘
              ↓
┌─────────────────────────────────────────┐
│         Evaluate alternatives           │
└─────────────────────────────────────────┘
              ↓
┌─────────────────────────────────────────┐
│         Choose best alternative         │
└─────────────────────────────────────────┘
              ↓
┌─────────────────────────────────────────┐
│       Implement the alternative         │
└─────────────────────────────────────────┘
```
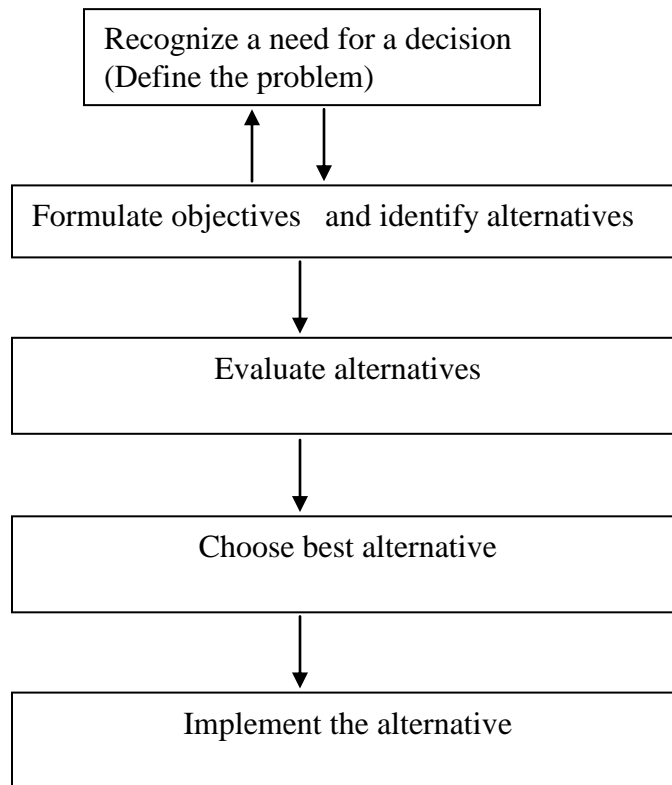
The process above has been recognized for along as a traditional approach to the decision-making process. The introduction of I.T has however led managers to challenge this approach. Opponents of this approach argue that no problem exists in Isolation and hence it cannot be considered for solution in isolation over the factors within the environment. Thus a total systems approach may be required to solve problems and make better decisions.

4.6   Conditions under Which Decisions Are Made

1) Decision making under certainty
In this case the decision maker has all the necessary information to make an optimum decision. Problems of this nature are solved through linear programming technique. Another technique is the analytical hierarchy process or approach – AHP.

2) Decision making under uncertainty

The uncertainty that surrounds many decisions arises from the lack of knowledge as to which state of nature will eventually occur. Thus decision making under uncertainty is the selection of an alternative action when the outcome depends on the chance occurrence of some state of nature that is outside the control of the decision maker.

Decision making under uncertainty is where the possible state of nature are known but their exact probabilities are unknown. Most business decision situations fall into this category.

3) Decision making under risk
Under this situation both the possible state of nature and their exact probabilities of occurrence are known.

4) Decision making under conflict
If the decision maker is pitted against an opponent it's said that the decision is made under conflict. In this case the probabilities are unknown since the response from the opponent is unknown. Decision-making under conflict is also known as "game theory" it is used in games such as chess and in military tactics.

5) Decision making under Ignorance
This is where neither the possible state of nature nor their exact probabilities are known. This is very rare as its unlikely that the decision maker will absolutely have no idea that about the state of nature that might occur after the decision has been made.

4.7   Mathematical Calculations

Decision Making Under Certainty
The analytical hierarchy process (approach) (AHP)
The structure of the decision problem involves hierarchy (level) criteria and decision alternatives (choices). The decision involves the determination of composite weights and the alternative with the highest composite weight is chosen.

Examples:

A student is to be sponsored to study in any of the three universities. University of A, University of B and University of C. To select the university s/he uses two criteria:

- Academic reputation.

- Cost.

Being in the university a student ranks academic reputation to be more important than cost. S/he assigns a weight of 83% to Academic reputation and 17% to cost. S/he then ranks the universities from the standpoint of academic reputation and cost as follows.

Cost: U of A (12.9%)
                 U of B (27.7%)
                 U of C (59.4%)

A. Reputation: U of A (54.5%)
                      U of B ((27.3%)
                      U of C (18.2%)

The structure of the decision problem is as below:

Decision: ⟶ [Select University]

Hierarchy:
(Criteria) ⟶ [Cost 0.17]   [Reputation 0.83]

Alternatives: ⟶   U of A U of B   U of C   U of A   U of B   U of C
(Choices)   0.129   0.277   0.594   0.545   0.273   0.182
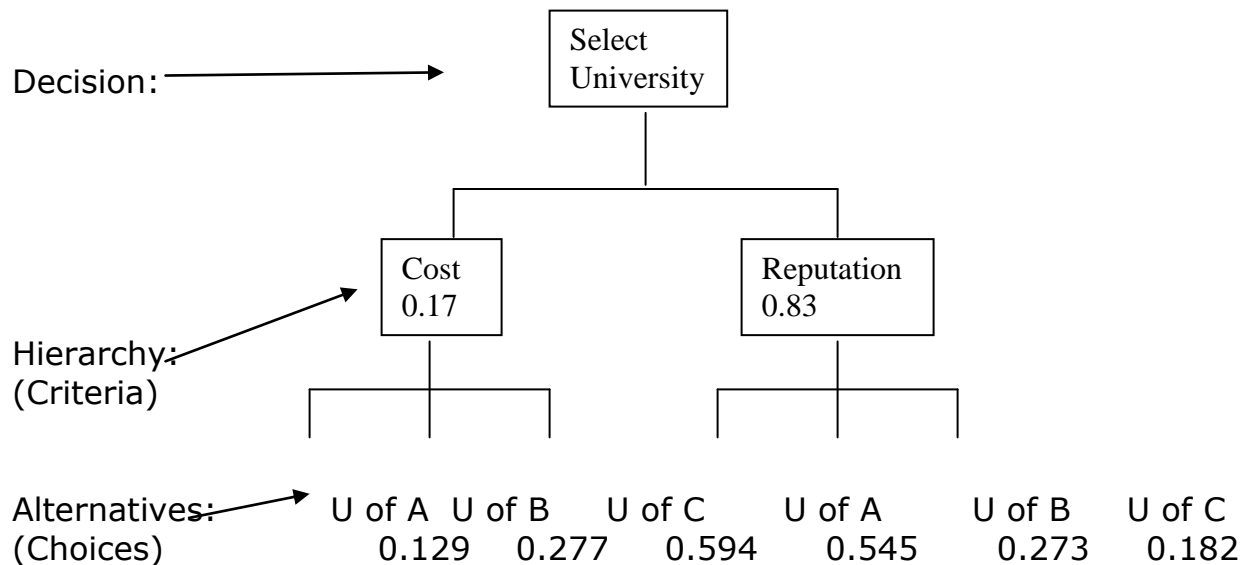
The composite weight for each university is:

U of A: (0.17* 0.129) + (0.83*0.545).

        = 0.4743

U of B: (0.17 * 0.277) + (0.83 *0.273).

        = 0.2737.

U of C: (0.12*0.594) + (0.83 * 0.182

$$= \underline{0.2520.}$$

A student will choose a university of A.

The general structure of AHP may include several hierarchies of criteria.
<u>Example.</u>

Suppose there are two students from the same family and they have to share a vehicle. Each of them in AHP

Ranks the universities according to the reputation and cost in the following decision structure.

<u>Example</u>

<u>Decision:</u> → Select a University

Hierarchy 1
Criteria

Student A — P

Student B — q

HIERARCHY
Criteria: ↗

Cost P_1

Reputation. P_2

Cost q_1

Reputation q_2

Alternatives

U of A U of B U of C
P_{11} p_{12} p_{13}

U of A U of B U of C q^{11} $q1_{12}$ q_{13}

U of A U of B U of C

q_{21} q_{22} q_{23}

U of A U of B U of C
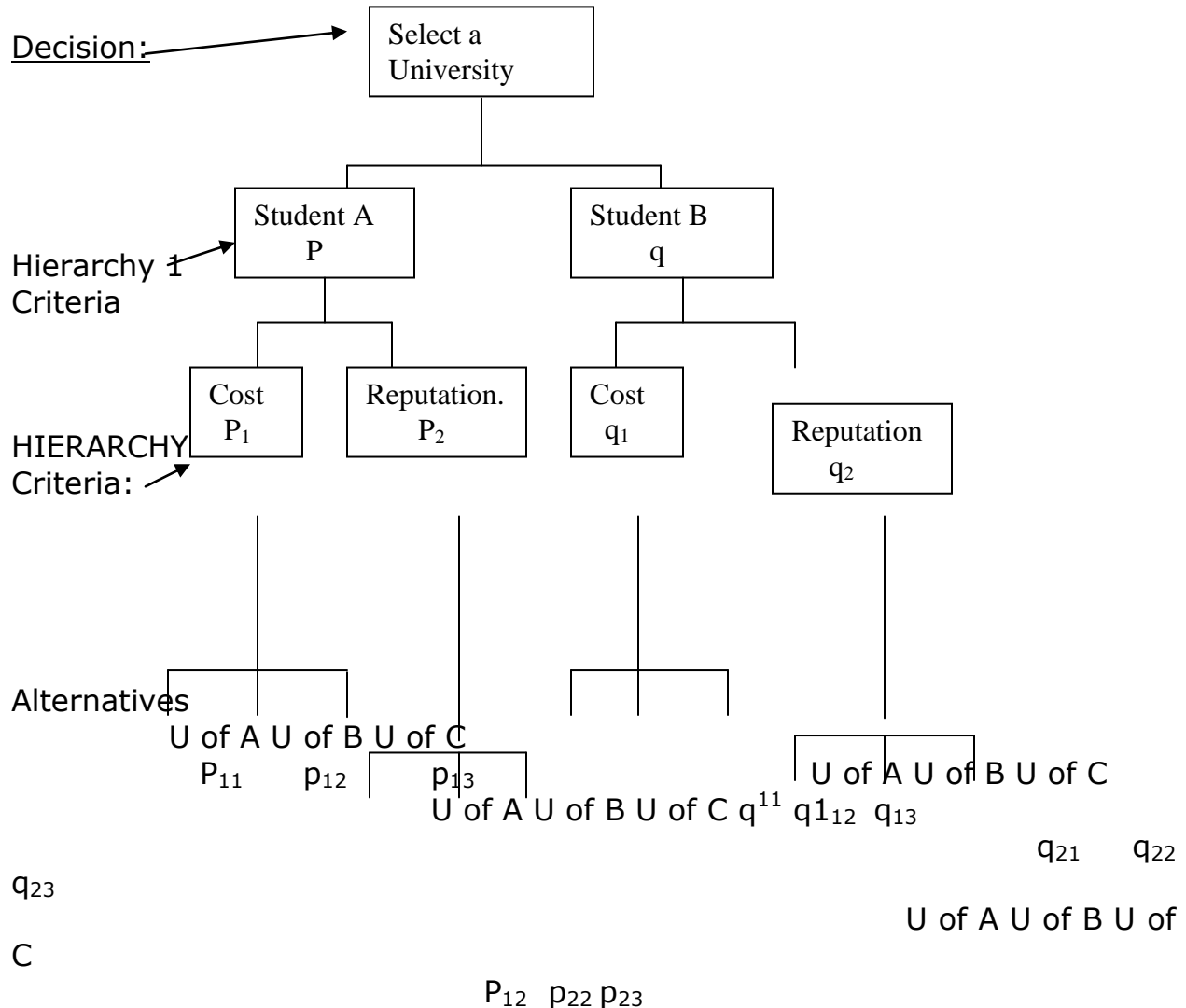
P_{12} p_{22} p_{23}

The composite weights for the alternatives are:

U of A = $(p_{11}*p_1*p) + (p_{21}*p_2*p) + (q_{11}*q_1*q) + (q_2*q_{22}*q)$.
        = $p (p_1p_{11}+p_2p_{21}) + (q_1q_{11}+q_2q_{21})$

U of B = $P (p_1p_{12} +p_2p_{22}) + q(q_1 q_{12} +q_2q_{22})$

U of C = $P (p_1p_{13} +p_2 p_{23}) +q (q_1 q_{13} +q_2 q_{23})$

Note:

    $P+q = 1$

    $P_1+p_2 =1$, $q_1+q_2 =1$.

    $P_{11}+p_{12}+p_{13} =1$, $p_{21}+p_{22}+p_{23} =1$.

    $q_{11}+q_{12}+q_{13} =1$, $q_{21}+q_{22}+q_{23} =1$.

Suppose we have the following weights

    $P= 0.5$, $q= 0.5$

    $P_1= 0.17$, $p2= 0.83$

    $P_{11}= 0.129$, $p12= 0.277$, $p13= 0.59$

    $P_{21}= 0.545$, $p22= 0.273$, $p23= 0.182$

    $q_1= 0.3$, $q2= 0.7$

    $q_{11}=0.2$, $q12= 0.3$, $q13= 0.5$

    $q_{21}= 0.5$, $q22= 0.2$, $q23= 0.3$.

U of A − 0.5 (0.17*0.129 +0.83*0.545) +
      0.5 (0.3*0.2 + 0.7*0.5.) +

  ❖ 0.5 (0.02193+0.45235) +0.5 (0.06+0.35)
  ❖   0.23714+0.205
  ❖   0.4421.

U of B = 0.5 (0.17*0.277 + 0.83*0.273) +

0.5 (0.3*0.3 +0.7*0.2)
        0.5 (0.04709 + 0.2266) + 0.5 (0.09+0.14)
   ❖   0.1368 + 0. 115
   ❖   0.2518

U of C = 0.5 (0.17*0.59 + 0.83*0.182) +

        0.5 (0.3*0.5 + 0.7*0.3)
        0.5 (0.10098 + 0.15106) +
        0.5 (0.15 +0.21)
   ❖   0.1258 + 0.15
        =   0.3058

Decision-Making Under Uncertainty
This depends on whether the decision maker wishes to adopt an aggression philosophy or follow a more conservative approach

Pay off table
This is a table in which the rows are decision alternatives, the columns are state of nature and the entry of each intersection of a row & column is a numerical payoff or outcome.

Example
Coca-cola intends to set up a bottling plant in East Africa. The state of the economy in each of the three East African countries is given. The relative profits in millions of dollars that are expected over a five year period is summarized in the payoff table below

Decision Alternative	State of Nature (Trend in Economy)		
	Decline	Expand	Stable
Uganda	- 1	5	3
Kenya	- 0.8	3.5	4
Tanzania	0.5	0.5	2

Optimistic Strategy
Under this strategy the decision maker adopts an aggressive optimistic strategy. S/he expects the best possible state of nature to occur. Under this strategy there are two decision criteria:

Maximum and Minimum Criteria

Maximum Criterion
This identifies the maximum payoff for each decision alternative and then selects the alternative that maximizes those maximums.

Decision	State of Nature Trends in the economy		
	Decline	**Expand**	**Stable**
Uganda	- 1	**5***	3
Kenya	- 0.8	3.5	4
Tanzania	0.5	3	2

Identify the maximum payoff for each decision alternative and choose the alternative yielding the highest of those maximums. Maximum involves choosing the highest payoff from the table. [i.e maximize the maximum]

The Maximum is used if the payoff are desirable variables or gains that we wish to maximize; such as profits, sales revenues, market share or rate of return

On the other hand if the payoff is undesirable such as costs, losses to or employee turnover is expressed as something we wish to minimize, we employ a minimum criterion.

Minimum Criterion

This is used when the payoffs are undesirable variables, which should be minimized, the lowest (best) payoff for each alternative decision is identified and the decision, which minimizes the minimum is selected [i.e minimizes the minimum]

Example

A grocery must decide on which banking option its bank offers that would result in lowest monthly charge. The bank has three alternatives.

Alternative A
A US $ 10 minimum charge plus US $ 4 cents per transaction

Alternative B
A US $ 11 minimum charge plus US $ 2 cents per transaction.

Alternative C
A flat rate of US $ 15 per month. Suppose you carryout 100, 200 and 300 transactions in month. What alternative would you choose?

	State of nature (transactions per month)		
Decision alternative	100 transactions	200 transactions	300 transactions
A	14	18	22
B	**13***	15	17
C	15	15	15

Under the Minimin criterion, you choose the decision alternative **B** and restrict transactions to **100**

Pessimistic Strategy
Under this strategy, it is assumed that given any decision alternative the state of nature that produces the worst possible results will occur. The rationale behind this strategy is to avoid large losses (even if it means the decision maker must settle for smaller gains).

Under this strategy there are two decision criterions: Maximin and minimax criteria.

Maximum Criterion
This criterion identifies the minimum (worst) payoff for each alternative and selects the alternative that maximizes those minimums. In other words the maximum criterion specifies that the decision maker selects the alternative having the highest minimum payoffs. i.e (this maximizes the minimums).

Decision	**State of nature (Trends of the economy)**		
	Decline	**Expand**	**Stable**

Uganda	- 1	5	3
Kenya	- 0.8	3.5	4
Tanzania	0.5*	3**	2

Minimum Criterion

This criterion identifies the highest payoff for each alternative and selects the decision that minimizes the maximums (i.e minimize the maximum)

It's used when we want to minimize undesirable payoffs.

-   We choose to invest in TZ when the economy is expanding.

Summary of strategy

Strategy	Desirable maximization	Undesirable maximization
Optimistic	Maximax	Minimin
Pessimistic	Maximin	Minimax

Opportunity Loss (Regret) Strategy

Is the difference between what is received given the decision made and what should have been received had the "right" decision been made. It's the lost value (regret) suffered by not making the correct decision. Of course if the optimum decision is made the loss or regret is zero. In other words the opportunity loss is the benefit foregone by not making the right decision.

Opportunity loss (for a cell) = maximum value in a column – sell value

Opportunity loss table for coca-cola

Decision Alternative	Decline	Expand	Stable
Uganda	1.5*	0	1
Kenya	1.3	1.5*	0
Tanzania	0	2*	2

The desire to minimize the loss (regret) calls for minimax criterion. The maximum regrets to build plants are 1.5 in Uganda, 1.5 in Kenya and 2 in TZ. The decision is to build either Uganda or Kenya from a strict interpretation however the decision will be to build in Uganda because it will result in only 1 million dollar regret if economy is stable compared to 1.3 million dollar in Kenya if the economy will decline in Kenya.

4.8   Decision Making Under Risk

When probability is assigned to the occurrence of each of the state of nature, it's said that decision is being made under risk. In such circumstances its possible to calculate the expected value of each decision alternative. The decision makers then select the alternative with high decision expected value. The expected payoff is sometimes referred to as expected monetary value (EMV).

 Our Expected value (payoff) - Ev = E xi Pcxi)
                                                   i= 1

Where xi is the payoff for the alternative and p(x) is the probability of the payoff (state of nature)

Decision	State of nature Trends in the economy		
Alternatives	Decline Prob. 0.3	Expanding 0.5	Stable 0.2
Uganda	- 1	5	3
Kenya	- 0.8	3.5	4
Tanzania	0.5	3	2

Expected pay off values
 E x P(x)
For Uganda, EMV
= (-1 x 0.3) + (5 x 0.5) + (3 x 0.2)
= - 3 + 2.5 + 0.6
= 2.8

For Kenya, EMV
(-0.8 x 0.3) + (3.5 x 0.5) + (4 x0.2)
= - 0.24 + 1.75 + 0.8
= 2.31
For Tanzania, EMV
(0.5 x 0.3) + (3 x 0.5) + (2 x 0.2)
= 2.05

The alternative in the highest payoff is Uganda with expected profit of 2.8 million dollars.

Decision Tree

Decision Alternative	States of nature	payoff	EV

```
                              0.3 decline        - 1
                                                              2.8
                      ( 1 )   0.5 Expand         - 5
                              0.2 Expand         - 3

                              0.3 decline        -0.8
   ( 0 )────────────( 2 )    0.5 expand          3.5          2.31

                              0.2 stable          4
                              0.3 Decline         0.5
                      ( 3 )   0.5 Expand          3            2.05

                              0.3 stable          2
```
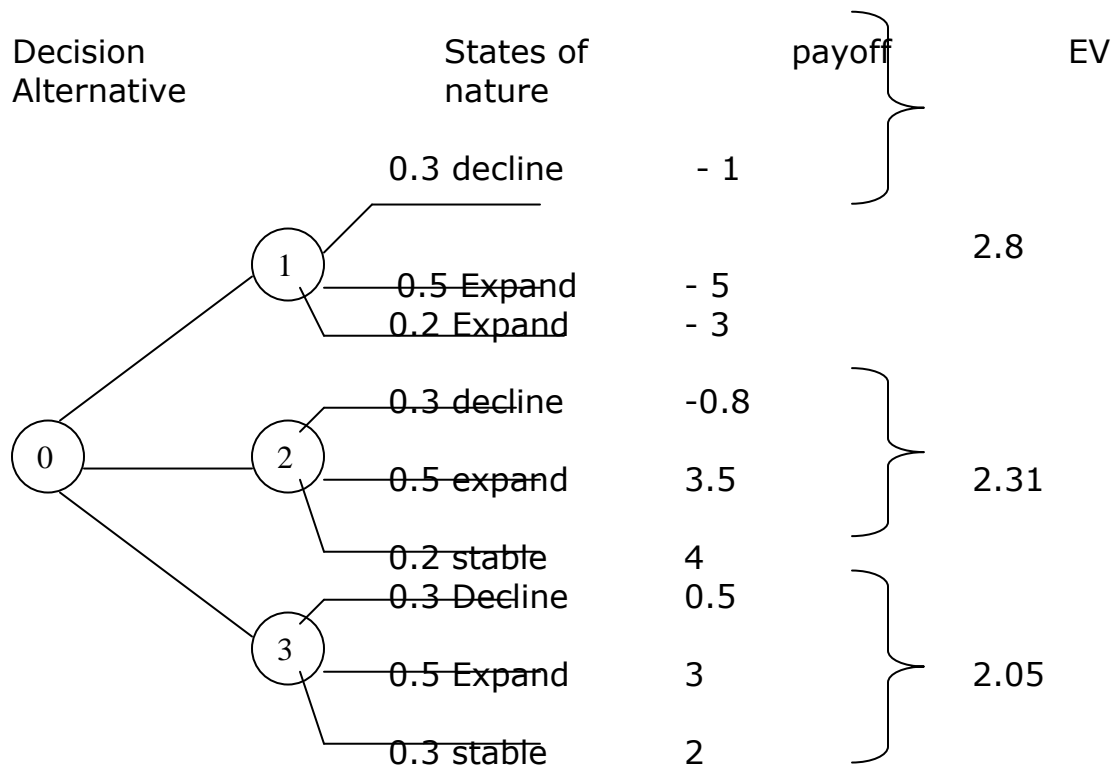
Review Questions

1. Explain the decision making process

2. Discuss the conditions under which decisions are made?

3. Explain the planning process

Unit 5
Inventory models and network analysis
5.1   Introduction
Inventory refers to goods that are in stock. These are goods waiting to be sold on the market or to be used in production of other goods, the goods that are used to manufacturer other products are called raw materials - While those waiting to be sold are called finished products.

Most business enterprises or industries usually maintain a reasonable inventory of goods to ensure smooth operations. Problems usually arise when too few or too many items are held in inventory. Traditionally inventory is viewed as a necessary evil because too little of it cause costly interruptions while too much result in idle capital. If too many items are held in inventory then:

- Capital (money) is tied up unnecessarily.
- Variable space is used unnecessarily
- Items may depreciate, deteriorate or become absolute.

On the other hand if too few items are held in inventory then:

- There may be a loss of profit because of loss of sales, which could lead to loss of goodwill due to unfilled demands.
- The manufacturing process may have to shutdown due to lack of raw materials.
- The cost to replenish inventory stock may be excessive when stock has to be ordered frequently.

A good inventory policy should be able to answer these two questions:
1. How much to order?
2. When to order

Basically the problem in inventory control is to minimize the total cost associated with maintaining an inventory.

These costs are:
1. **Purchasing cost**. This is based on the price per unit of the item. It may be constant or it may be offered at a discount depending on the size of the order.

2. **Set up cost (Ordering cost).** This represents the fixed charges whenever an order is placed. Its independent of the size of the order.

3. Carriage cost or holding cost.

This represents the cost of maintaining the inventory in stock. It includes the interest on capital as well as the cost of storage, maintenance and handling.

4. Shortage cost
This is the penalty cost incurred when the stock runs out, it includes potential loss of income as well as the more subjective cost of loss of customer's goodwill.

Total inventory cost
T.I.C = Purchasing cost + set up cost + holding cost + shortage cost.

5.2   Economic Order Quantity (Eoq) Model

This is the value of the quantity ordered that minimizes the total inventory cost. This EOQ model assumes the following:

1. The demand is known and it is constant.
2. The concept time of order is instantaneous i.e the order is received in batch.
3. The lead-time i.e the time between the requisition and receipt of an order is known as constant.
4. There are no quantity discounts.
5. There are no stock-outs (shortages)

Let Q = order quantity (the amount to order each time an order is placed).
    D = Demand rate for the product
    K = Ordering cost per order (set up cost) (cost to place on order)
    B = Inventory holding cost per unit (this is the cost of holding one unit of stock.

We can derive the following:

(1) Average Inventory level =
    Quantity ordered / 2 = $Q/2$

2. Number of orders per time period
    = D/Q

3. Ordering cost (OC) = $K D/Q$

(4) Inventory holding cost (IHC)
    = $h \underline{Q}$

2

(5) Ordering cycle (to) = Q/D
Total inventory cost (TIC)
TIC = OC + IHC
= K. D/Q + H. Q/2

TC = Purchasing cost + OC + IHC

EOQ minimizes TIC
TIC = K D/Q + H. Q/2

EOQ is the value of the order quantity Q that minimizes total inventory cost. In order to get EOQ we differentiate total inventory cost (TIC) with respect to Q and set the results to Zero and then solve for Q. Alternatively we equate OC to IHC i.e EOQ is that quantity where

OC = IHC.

We have
$$OC = K. D/Q, \quad IHC = h. Q/2$$

$$K. D/Q = H. Q/2$$

$$\frac{2\ KD}{h} = Q^2$$

$$\frac{1}{\sqrt{\dfrac{2KD}{h}}} = Q$$

$$\left(2KD \right)^{\frac{1}{2}} = Q$$

Where
D = Demand
K = Ordering cost
H = Holding (carriage cost)

Example

Light Bulbs at KIU are replaced at a rate of 100 per month. It costs 100/= to initiate a purchase. The storage cost of a bulb 2/= per month. Determine the optimal inventory policy for ordering bulbs by KIU

CC = 100/= per order

AC = 2/= per month

D = 100 per month

RE-ORDER POINT

A new order need not be received at the instant it's ordered. A lead-time (L) may occur between the requisition and receipt of an order. In this case the re-order point occurs when the inventory level drops to LD units. A good system assumes that the lead-time is less than the cycle time, i.e $L < t_0$

$t_0 = Q/D =$ 100 bulbs
            100 bulbs/month
         = 1 per month

Suppose the lead-time to receive bulbs is 3 days what is the re-order point?

Re-order Rt = LD

D = 100 per month
L = 3 days

D = 100/30   = 10/3 per day.

LD = 10/3 x 3 days
LD = 10 bulbs

5.3   Types of Networks ANALYSIS

Network Models (Techniques)
PERT and CPM are two popular quantitative analysis techniques that help managers plan, schedule, monitor and control large and complex projects. Other network techniques are:
- Minimal spinning tree technique
- Maximal flow techniques
- Shortest Root technique

PERT Vs CPM
For every PERT activity, three time estimates are combined to determine the exported activity completion time and its variance. Thus PERT is a

probabilistic technique. It allows us to find the probability; the entire project will be completed by a given data. CPM on the other hand is called Deterministic approach. It uses two time estimate, the normal and crash time for each activity. A normal completion time is a time on estimate it would take under normal condition to complete the activity and the crash completion time is the shortest time it would take to finish the activity of additional funds and resources were allocated to the task.

Tasks Common to Both Pert and Cpm
1) Define the project and all its significant activities or task.
2) Develop the relationships among the activities. Decide which activities precede and follow others.
3) Draw the network connecting all the activities.
4) Assign time for cost estimate to each activity.
5) Compute the largest time path through the networks; this is called the critical path.
6) Use the networks to help plan, schedule, and monitor and control the project.

Finding the critical path is a measure part of controlling the project the activities on the critical path represent tasks that will delay the entire project id they are delayed.

Let us now consider each of these methods in greater detail.

5.4   PERT

Definition: A network consists of a set of nodes   linked by arcs (or branches).
An activity comes the arrow or (arc) symbol. This represents a task or such project that uses the time or resources. An event marks the start or completion of a particular activity its noted by a node or symbol O. This contains a number that helps to identify its location.

Example
Suppose a firm is to install an air pollution filter the project is to last for sixteen weeks. Below are the activities involved in this project?

Step 1: Map out activities

Activity	Description	Prioritize (step 2) Immediate predecessors

A	Build internal components	-
B	Modify the roof and floor	-
C	Construct a collection stance	A
D	P our concrete and install frame	B
E	Build a high temperature burner	C
F	Install control system	C
G	Install air pollution device	D & E
H	Inspection and testing	F and G

Step 3 Draw a net work
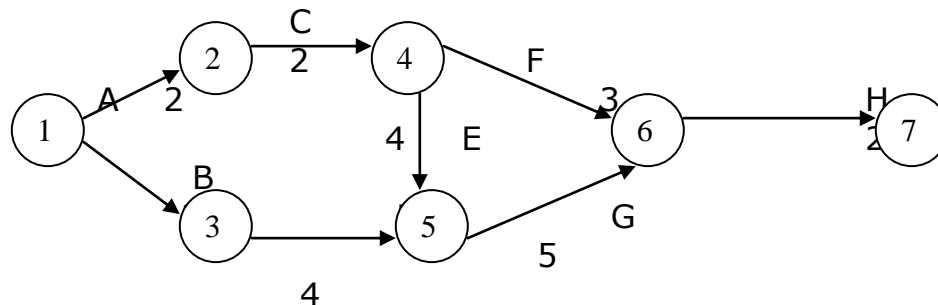
Step 4. Assign time estimates required to complete each activity. There are 3 time estimates that are assigned to each activity.

These are:
  (i)   **Optimistic time** (a): This is the time   an activity will take if everything goes as well as possible.
  (ii) **Most likely time** (M): This is the most realistic time estimate to complete the activity.
  (iii) **Pessimistic time** (b): this is the time an activity would take assuming very unfavorable conditions.

The expected time of completion of an activity follows a beta probability distribution when expected time of completion of an activity (t) is

$$T = \frac{At\ 4m + b}{6}$$

The variance of the expected time estimate is

Variance $= \frac{(b - a)^2}{6}$

Suppose we have the full time estimates in week in our example

Activity	Optimistic (a)	Mostly likely (M)	Pessimbistic (b)
A	1	2	3
B	2	3	4
C	1	2	3
D	2	4	6
E	1	4	7
F	1	2	9
G	3	4	11
H	1	2	3

	Expected time $T = \frac{a\ p4m + b}{6}$	Variance $(\frac{b-a}{6})$
A	2	4/36
B	3	436
C	2	4/36
D	4	16/36
E	4	36/36
F	3	64/36
G	5	64/36
H	2	4/36

Step 5: Determine the critical path:
This is the largest time path root through the network. Any delay of an activity on the critical path would delay completion of the entire project.

To find the critical path, we need to determine the following quantities for each activity in network.

(A) The earliest start time (ES):
This is the earliest time an activity can begin without violation of immediate predecessor requirements

(B) The earliest finish time (EF):
This is the earliest at which an activity can end.

(C) Latest start time (LS)
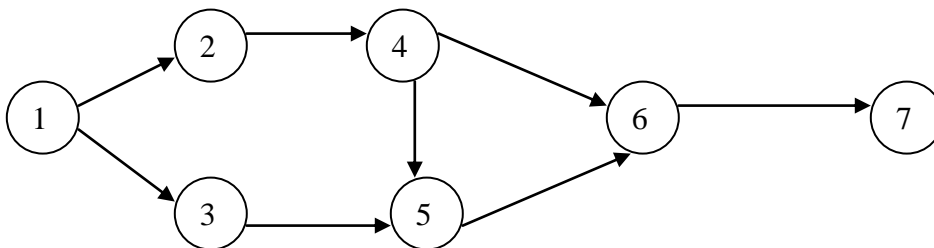This is the latest time an activity can begin without delaying the entire project.

(D) The latest finish time (LF)
This is the latest time an activity can end without delaying the entire project.
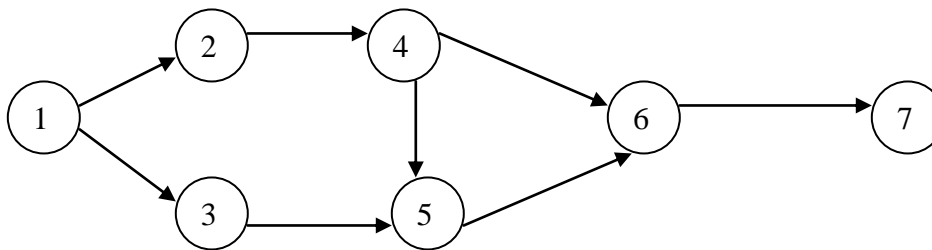
EF – ES + t
Where t = Expected activity time

Before any activity can be started all of its predecessors activities must be completed.

- Computer the latest start time LS = LF - t

Slack time: This is the length of time an activity can be delayed without delaying the whole project.

Stock =   LS   –   ES

OR

LF   -   EF

Activity	Es	EF	LS	LF	Slack
A	0	2	0	2	0
B	0	3	1	4	1
C	2	4	2	4	0
D	3	7	4	8	1
E	4	8	4	8	0
F	4	7	10	13	6
G	8	13	8	13	0
H	13	15	13	15	0

Activities that have no slack time are said to be critical activities. Because of this, critical activities are said to be on the critical path. In the previous example activities A, C,E,G & H are critical activities and the path A   C   E   G   H is the critical path.

Probability Analysis of Project Completion

Critical Activity	Expected time of completion	Variance
A	2	4/36
C	2	4/36
E	4	36/36
G	5	64/36
H	2	4/36
TOTAL	15	112/36

Project variance = sum of variance of activities on the critical path.

Project standard deviation
= Project variance
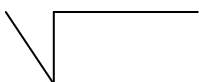
=   112/36   =   3.11

                                        = 1.76weeks

PERT makes two assumptions

(1) The total project completion times follow a normal probability distribution.
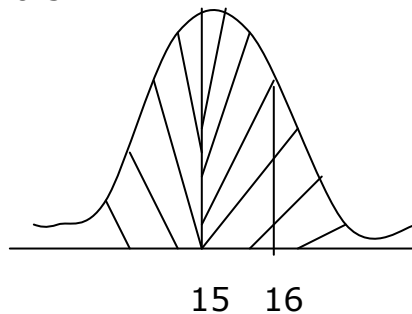(2) The activity times are statistically independent.

To find the probability that the project will be finished on or before the deadline, to determine the approximate area under the normal curve by applying standard normal equation.

$Z = \dfrac{\text{Due time} - \text{Expected time of competition}}{\delta_1}$

$= \dfrac{16 - 15}{176}$ =   0.568

                    = 0.57

                     15   16

       0.5000 + 0.2157 =   07157

The probability that the project will be completed on time is 0.716 or there is 71% that the project will be completed on time.

PERT COST
Pert is used to monitor and control project length. However, it doesn't consider another very important factor, project cost. Pert/cost is the modification of pert that allows the manager to plan, monitor and control and schedule cost as well as time.

PERT IN ACTION
The overall approach in the budgeting process of a project is to determine how much is to be spent in every time period (weeks or months or years).

This is accomplished by the following steps:

Step 1: Identify all costs associated with each of the activities. The add these costs to get one estimated cost or budget for each activity.

Step 2: If you are dealing with a large project, several activities may be combined into larger work packages. Work package is the logical collection of activities.

Step 3: Covert the budgeted cost per activity into a cost per time period. To do this, we assume the cost of completing any activity is spent at a uniform rate over time for example if the budgeted cost for a given activity is $ 40,000 and the activity's expected time is 4 weeks, the budgeted cost per week.

$ 40,000 = $ 10,000 per week.

Step 4: Using the earliest and latest start times, find out how much money should be spent during each time period, in order to finish the project at the desired date.

Activity	Es	Ls	Expected time (t)	Total budgeted cost (Us $)	Budgeted cost per week Us $
A	0	0	2	22,000	11,000
B	0	1	3	30,000	10,000
C	2	2	2	26,000	13,000
D	3	4	4	48,000	12,000
E	4	4	4	56,000	14,000
F	4	10	3	30,000	10,000
G	8	8	5	80,000	16,000
H	13	13	2	16,000	13,000
Total				308,000	99,000

Budgeted cost using Es times (Cost in 000 US $)
Week

Activity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	11	11													
B	10	10	10												
C			13	13											

Activity	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D				12	12	12	12								
E					14	14	14	14							
F					10	10	10		16	16	16	16			
G															
H														8	8
Total/wk	21	21	23	25	36	36	36	36	14	16	16	16	16	8	8
Total to date	21	42	65	90	126	162	198	212	228	244	260	276	292	300	308

Those activities on the critical path must ….. their budgets Es time. The activities that are not in the critical however can be started at a later date (latest start date).

Budgeted cost using Ls times (cost in 000$)
Week

Value of work completed
The value of work completed or cost to date for any activity can be computed as follows:

Value of work completed =1% of work completed x total activity budget.
Activity difference = Actual cost – value of work completed, if an activity difference is negative, there is a cost under non but if a number is positive, there is a cost overrun. To monitor and control costs, a budget amount a value of work completed and the actual cost should be completed periodically.

5.5 Critical Path Method (Cpm)

CPM is a deterministic network model, this means that both the time and cost to complete each activity are known with certainty.

CPM uses two sets of time and cost estimates for activities. A normal time and cost and crash time and cost. The normal time is the time we estimate it will take under normal conditions to complete the activity.

It is like PERT's expected time. Normal cost is an estimate of how much money it will take to complete an activity in its normal time.

The crash time is the shortest time it   would take to finish an activity if additional funds and resources were allocated to the task. It's the shortest possible activity time. Crash cost is the price of completing the activity on the crash or deadline basis.

Project Planning with CPM
Project planning with CPM involve the following steps.
1) Find the critical path and identify the critical activities. This is done by following the same steps as used in pert's Es, Ls, EF and LF times and slack.
2) To compute the crash cost per time period for all the activities in the network.

   Crash cost / time period = $\dfrac{\text{Crash cost – Normal cost}}{\text{Normal time – Crash time}}$

3) Select the activity on the critical path with the smallest crash cost per time period. Crash this activity to the maximum extent possible or to the point at which the desired deadline has been reached.
4) Check to be sure, the critical path on your crashing is still critical. Often reduction in activity time along a critical path causes a non-critical path or paths to become critical. If the critical path is still the longest path through the network, return to step 3. If not, find the new critical path and then return to step 3.

Minimal Spanning Tree Techinque
Seeks to connect all the points of a network together while minimize the distance between them. It has been used when connecting or supplying electricity, water, telephone lines e.t.c services to houses.

It seeks to minimize a total length of cables or pipes.

Minimal spanning tree technique involves the following steps:

1. Select any node in the network
2. Connect this node to the nearest node that minimizes the total distance
3. Considering all of the nodes that are now connected, find and connect the nearest node that is not connected.
4. Repeat step 3 until all nodes are connected.
5. If there is a tie a in step 3 and 2 or more nodes that are not connected are equally near arbitrarily select one and continue.

Example
Suppose there are eight houses that need to be supplied with water. The distance between each house (in hundreds of feet) is shown on the network.
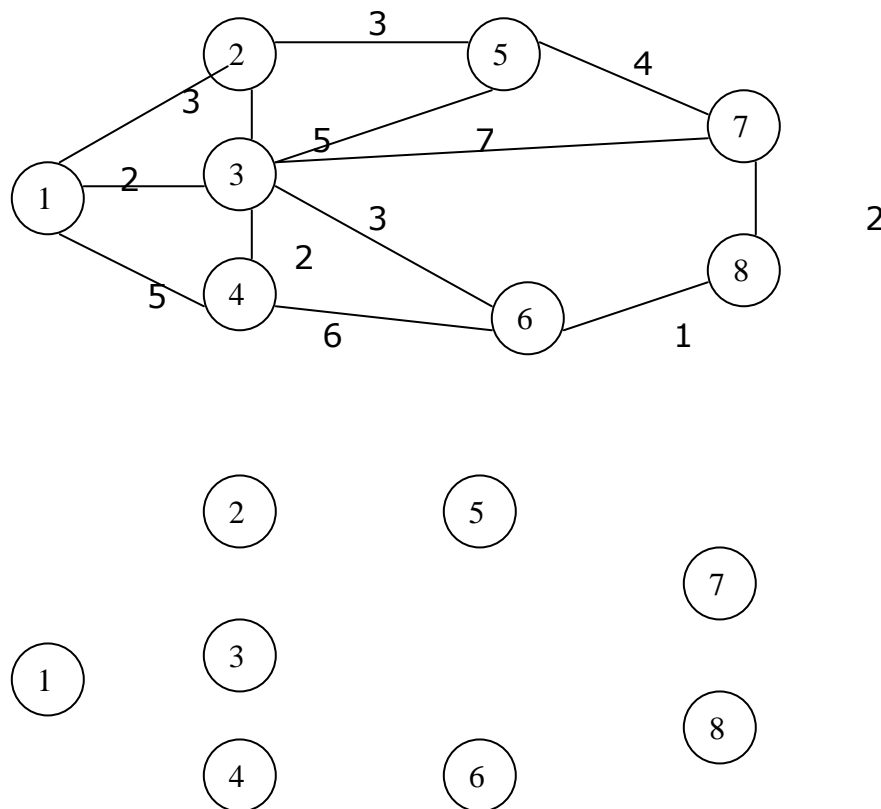
Note: The order in which connection are made is not important

Maximum Flow Techniques

This technique allows us to find the maximum flow of any quality or substance through a network. This technique can be used to determine the number of vehicles that can go through a net work of roads from one location to another.

This technique involve the following steps

1. Pick any path with some flow.
2. Increase the flow as much as possible.
3. Adjust the flow capacity numbers in the path.
4. Repeat the above steps until an increase in flow is no longer possible.

Example
Suppose we have the following road network in a small town. The number of the nodes indicates the maximum number of cars (in hundreds of cars per hour) that can flow from the various nodes.
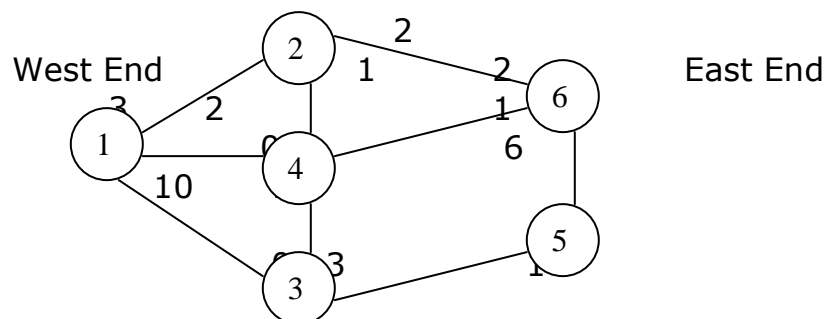

Shortest Route Technique
This technique is used to find the shortest not through a network for example this technique is used to find how a person or item can travel from one occasion to another while minimizes the total distance traveled. This technique involves the following step.

1. Find the nearest node to the origin
2. Find the next nearest node to the origin. In some cases several path will have to be checked to find the nearest node.
3. Repeat this process until you have gone through the entire network. The last distance at the ending node will be the distance at the shortest root.

Example: Suppose a manufacturer has to transport mercantile from the plant to the warehouse. Below are the routes and distance in KMs that have to be traveled.
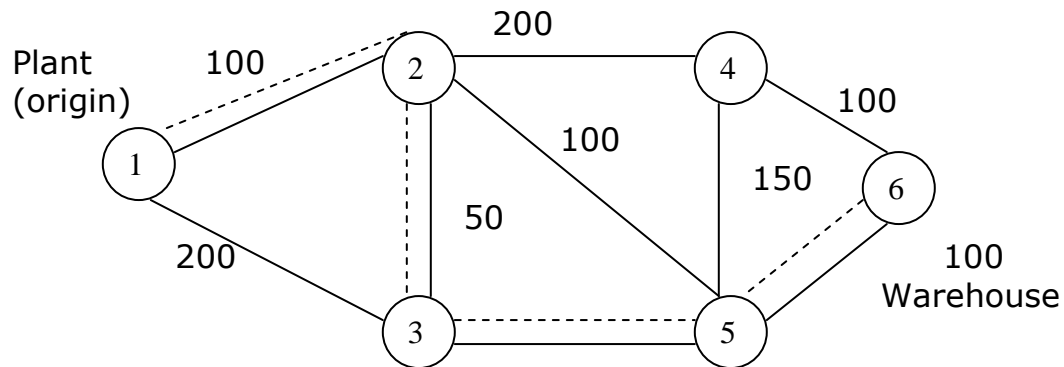
The shortest root is 1 – 2 – 3 – 5 – 6 and the total distance covered is 100 + 50+40+100 = 290kms.

Unit 6
Matrices and their palliation
6.1   Matrix Algbra (Matrices)

It deals with more than one number at a time. Any collection of numbers is known as a set and matrices help us to handle arrays. Each unit in the array is called an element.

Notation
A vector is denoted by small letters printed broadly or simply as that small letter underlined for example $a = a$ a_1

$$\begin{pmatrix} a_2 \\ a_n \end{pmatrix}$$

and this is called a column vector

$C = C = \left\{ \quad C_1, C_2 \text{.....................} \right\} n$ and this is a row vector.

So in numbers

$a = a$

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$ column vector

$C = C = \quad 2, 3, 4$ Row vector

Transposing a matrix
Turning a row matrix into a column vice versa, it is known as transposing a matrix

For example given $a = $ $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ then the transpose of a which is
donated by a^1 (transpose) $a_1 = a_1, a_2, a_3$ changing a column matrix to a row matrix

Example given $P = \quad 2, 5, 9$

Find $P_1 = \begin{pmatrix} 2 \\ 5 \\ 9 \end{pmatrix}$

Equality

The vectors X and Y, X & Y will equal if they have the same number of elements.

Dimension. This refers to the number of rows by column. It is also refered to as the order of the matrix.

E.g.

$$A = \begin{pmatrix} a_{11}, & 9_{12} & a_{13}............a_{1n} \\ A_{22} & ....a_{2n} \\ A_{m1}..................a_{mn} \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 4 & 5 \\ 6 & 4 & 9 \\ 7 & 8 & 4 \end{pmatrix}$$

Given the matrix earlier, then the matrix m is of order m x n

$$\text{Suppose } ß = \begin{pmatrix} 2 & -1 & 10 \\ 3 & 5 & ½ \end{pmatrix}$$

Then the order of this matrix is a 2 X 3.

6.2   Types of Matrices

1. Square matrices. It is where the number of rows are equal to columns. Considering above matrix A, it is where m = n.

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   2 x 2

2. Diagonal matrices. It is where all elements are equal to zero apart from the diagonal elements.

$$\text{e.g.} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

3. Null matrices. It is where all elements are equal to zero.

4. Unit / Identity matrices. All the elements along the principle diagonal are equal to one.

5. Scarter matrix. It is a one by one matrix

W   =   [R)

Matrix Operations

a)   Additions and subtractions
Given that $Ç = A + ß$
$Ç = A + ß$
m x n   m x n the order should be the same

Such that $Çi] = A_{i]} + ß_{i]}$

e.g. if $Ç_{1x\,3} + A_{1x3} + ß_{1x3}$

NB we can only add matrices of the same order. In other words

e.g. $A = \begin{pmatrix} 2 & 4 \\ 6 & 7 \end{pmatrix}$ $B = \begin{pmatrix} 6 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix}$ you can't add this
because
they are not of the
same
order.

If $Ç = A + ß$, then similarly $Ç = ß + A$
Use the bodimus. $A + ß + Ç = (A + ß) + Ç$

e.g. $(A + ß)_1$ it is the same as $A^{\pm} + ß^{\pm}$

b) MULTIPLICATION

Given $X_1, X_2, X_3 \ldots\ldots\ldots\ldots X_n$

As quantities and P_1, P_2, P_3 …….as unit prices, then the value of the goods.

Value $= P1 + P_2 x\, X_1 + P2x\, X3 + P_3 x\, X_3$

Is the same as $\sum^n P_i X_i$
           $i = 1$
i is moving from I to n

x moves from 1 to m.

This can be represented in matrix form as follows

$$[p_1\ p_2\ p_3\ p_4] \qquad \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

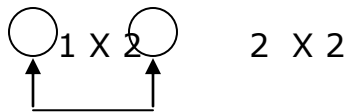Compatibility of matrices

Two matrices are comparable if

$A mn\ =\ \quad A m_1 n_1$
$ß mn\ =\ \quad B m_2 n_2$ $\qquad\qquad$ Such that $n_1 = n_2$

$A\ ß \qquad\qquad A$ $\qquad\qquad$ then the order or circled elements = 1
$\qquad\qquad\qquad\qquad\qquad n_1$ & m_2 must be the same.

$A \qquad = \begin{bmatrix} 2 & 5 \end{bmatrix}$ $\qquad\qquad\qquad A\ X\ ß$

$ß\ = \begin{bmatrix} 4 & 6 \\ 7 & 8 \end{bmatrix}$ $\qquad\qquad \bigcirc 1\ X\ 2 \bigcirc \qquad 2\ X\ 2$

$\qquad [\ 2\ 5] \qquad\qquad\qquad \begin{bmatrix} 4 & 6 \\ 7 & 8 \end{bmatrix}$

The out come for the order should always be $m_1 n_2$ $\qquad\qquad$ 2 x 4 x 3 x 7 =
43

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 8 + 35
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ [43   52] it is a 1 x 2
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 2 x 6 + 5 x 8 $\qquad$ = 52
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 12 $\qquad$ 40

Example 1

Given that $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 3 & 1 \end{pmatrix}$
$\qquad\qquad\qquad$ 2 x 3

$ß = \begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & 1 \\ 1 & 5 & 2 \end{pmatrix}$
$\qquad\qquad\qquad$ 3x3

First find the order of the matrix
Find
    (i)   A + ß
    (ii)   ß + A
    (iii) ß - A
    (iv) Aß
    (v)   ßA
    (vi) A - ß

Answers

    (i)   A + ß it is incompatible
    (ii)   ß + A it is incompatible
    (iii) ß - A it is Incompatible
    (iv) Aß Compatible
    (v)   ßA it's Incompatible
    (vi) A - ß it's Incompatible

    (vii) Aß $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 3 & 1 \end{pmatrix}$ $ß = \begin{pmatrix} 0 & -1 & 3 \\ 2 & 4 & 1 \\ 1 & 5 & 2 \end{pmatrix}$ $= \begin{pmatrix} 4 & 19 & 11 \\ 7 & 15 & 11 \end{pmatrix}$

Example 2

2. Given $a = [2\ 4\ 6]$ $ß = \begin{matrix} -2 & -1 \\ 4 & 4 \end{matrix}$

            $b = [1\ 5\ 9]$ $A = \begin{pmatrix} 4 & 5 \\ -1 & 0 \end{pmatrix}$ $D = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 5 & 10 \\ 1 & 8 & 10 \end{pmatrix}$

                  $C = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 5 & 7 \\ 1 & 5 & 8 \end{pmatrix}$

Find   (i) a + b
           (ii) $A^1 + ß^1$
           (iii) $ß^1 + A^1$
     (iv) $ß + A^1$
     (v) $A + ß^1$
    (vi) $(a^1 + b^1)$
    (vii) $(A + ß)^1$
    (viii) $a + b^1$ incompatible
    (ix) a^1 b Incompatible
    (x) a C Incompatible

(xi) AB
Show that Aß = ß¹A₁¹

Solution

(i) a + b   a¹ = [2 4 6]   b¹ = 1 5 9

$$a = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 5 \\ 9 \end{bmatrix}$$

$$a + b = \begin{bmatrix} 3 \\ 9 \\ 15 \end{bmatrix}$$

(ii) A₁¹ + ß¹   = ß = $\begin{bmatrix} -2 & -1 \\ 4 & 4 \end{bmatrix}$ A₁¹ = $\begin{bmatrix} 4 & -1 \\ 5 & 0 \end{bmatrix}_{2 \times 2}$ + ß¹ = $\begin{bmatrix} -2 & -1 \\ -1 & 4 \end{bmatrix}_{2 \times 2}$

(iii) ß¹ - A₁¹   =   ß¹ = $\begin{bmatrix} -2 & 4 \\ -1 & 4 \end{bmatrix}$ A₁¹ = $\begin{bmatrix} 4 & -1 \\ 5 & 0 \end{bmatrix}$ A₁¹ + ß¹ = $\begin{bmatrix} 2 & 4 \\ 2 & 9 \end{bmatrix}$

ß¹ - A₁¹ = $\begin{bmatrix} -6 & 0 \\ -1 & 4 \end{bmatrix}$ = ß¹ - A₁¹ = $\begin{bmatrix} 0 & -1 \\ -6 & 4 \end{bmatrix}$

(iv) ß + A₁¹ = $\begin{bmatrix} -2 & -1 \\ 4 & 4 \end{bmatrix}$ + A₁¹ = $\begin{bmatrix} 4 & -1 \\ 5 & 0 \end{bmatrix}$ = $\begin{bmatrix} 2 & 5 \\ -4 & 4 \end{bmatrix}$

(v) A₁¹ - ß¹ = A = $\begin{bmatrix} 4 & 5 \\ -1 & 0 \end{bmatrix}$ - ß¹ = $\begin{bmatrix} -2 & 4 \\ -1 & 4 \end{bmatrix}$ = $\begin{bmatrix} 6 & -5 \\ 6 & -4 \end{bmatrix}$

(vi) a¹ + b¹ = [2 4 6] + b [1 5 9]

A = $\begin{bmatrix} 4 & 5 \\ -1 & 0 \end{bmatrix}$ x ß = $\begin{bmatrix} -2 & -1 \\ 4 & 4 \end{bmatrix}$

(vii) A + ß¹   A = $\begin{bmatrix} 4 & 5 \\ -1 & 0 \end{bmatrix}$ + ß¹ $\begin{bmatrix} -2 & 4 \\ -1 & 4 \end{bmatrix}$

= $\begin{bmatrix} -8 & -4 \end{bmatrix}$

$$\begin{array}{cc} -5 & 0 \end{array}$$

(ix) $A_t + ß = \begin{bmatrix} -8 & 1 \\ 20 & 0 \end{bmatrix}$

Example 4

Textile limited specialists in men's wear, ladies' wear and children wear operates 3 branches in Kampala, Masaka and Kabale. Purchasing of the 3 products and costing is decentralized and hence prices are uniform at the 3 branches. Sales pre week, unit prices and unit costs are as follows:-

	Gents	Ladies	Children
Kampala	67 units	424 units	18 units
Masaka	85 units	704 units	27 units
Kabale	48 units	264 units	12 units

Costs per unit   selling price pre unit.

G.w   shs 1600   G.w   shs 1800

L.w   shs 1050   L.w   shs 1200

C.w   shs 480   C.w   shs 600

Required
1) Using appropriate matrix operations, determine the company's weekly profits for all branches.

Profits = TR – TC.

Example 5

Roko Construction Company has got a contract to construct facilities at Mbarara University of Science and Technology (Must). The facilities required are 5Presidential houses, 7 students' halls and 12 classes. The R/m^s that are required for each type of house are wood, bricks, Glass, paint and Labour. The R/m^s are required in the following quantities.

	Wood	Bricks	Glass	Paint	Labour		15,000	wood
5 Resident	5	20	16	7	17		8,000	bricks
7 Hall	7	18	12	9	21		5,000	Glass
12 Class room	6	25	8	5	13		1,000	Paint
							10,000	Labour

The cost of Raw materials is given Shs 15,000 per unit of wood 8000 per unit of bricks, 5000 per unit of Glass, 1000 per unit of paint and 10,000 per unit of labour.

Required

(i) The amount of Raw materials required to complete the contract.
(ii) The cost of each type of building.
(iii) The total cost of Raw materials for the whole contract

Raw materials

	W	B	G	P	L
5	5	20	16	7	17
7	7	18	12	9	21
12	6	25	8	5	13

Costs

W	-	15000
B	-	5000
G	-	5000
P	-	1000
L	-	10000

(ii) [5 7 12] X

W	B	G	P	L
5	20	16	7	17
7	18	12	9	21
6	25	8	5	13

The amount of Raw materials required to complete the contract

W	146
B	526
G	260

P   158
L   388

(ii) Cost of each type of building   Cost of each type of building

$$
\begin{array}{l}
W \\ B \\ G \\ P \\ L
\end{array}
\begin{pmatrix}
146 \\ 526 \\ 260 \\ 158 \\ 388
\end{pmatrix}
\qquad
\begin{array}{l}
 \\ \\ \text{Costs} \\ \\
\end{array}
\begin{array}{l}
W \\ B \\ G \\ P \\ L
\end{array}
\begin{pmatrix}
15000 \\ 8000 \\ 5000 \\ 1000 \\ 10,000
\end{pmatrix}
\qquad
\begin{array}{l}
W \\ B \\ G \\ P \\ L
\end{array}
\begin{pmatrix}
219000 \\ 4208000 \\ 130000 \\ 158000 \\ 3880000
\end{pmatrix}
$$

(iii) The total cost of raw materials for the whole contract = 10566000

The Concept Determinant

If A =

Which is a 2 x 2 matrix, the determinant is

$|A|$ = a_{11} a_{12} − a_{21} a_{12}

β =

The determinant is given by

$|\beta|$ = (2 x 7) − (5 x 4)
      14 − 20
      -6

So, in a two dimensional matrix such as

The determinant is given by the difference into the product of the two diagonals starting from left to right. The same approach is true for all matrices even though we may give a different consideration in the classification of the diagonals.

NB: Only square matrices have determinants apart from the null matrix whose determinant is Zero.

e.g.   A $\begin{pmatrix} 2 & 5 \\ 3 & 67 \end{pmatrix}$

The Minor

A minor is a sub matrix derived from a matrix.

e.g. A = $\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix}$ $\begin{pmatrix} a22 & a23 \\ a32 & a33 \end{pmatrix}$

The Con-factor Matrix ([3 X 3] matrix) read about it.
It is a determinant based on the minor.
The adjoin matrix
Each square matrix has an adjoin matrix. The ad-joint matrix is the con-factor matrix transposed it is donated by $(A^{+})^{1}$

Review Questions

1. URA started operations in September 1991. It is contracted ROKO construction to rehabilitate its new headquarters at Nakawa. Roko has surveyed the place and found that the place and found that the work involved construction of 4 long rooms, 8 offices and 5 board rooms. The units of raw materials required per unit as follows:

	Steel	Wood	Glass		Paint	Labour
4 long rooms	7	15	16	20	17	
8 office	2	5	8	9	21	
5 Boardroom	5	11	14	7	13	

Costs of the raw materials per unit were

Raw materials   Shs

Steel	7,000
Wood	16,000
Glass	3,500
Paint	6,000
Labour	15,000

Required

(i) What amount of each type of Raw material should ROKO order?
(ii) What is the cost of Raw materials for each type of facility?
(iii) What is the total Raw materials costs for the whole job?

Unit 7
Index numbers

7.1   Introduction

An **index number** is an economic data figure reflecting price or quantity compared with a standard or base value. The base usually equals 100 and the index number is usually expressed as 100 times the ratio to the base value. For example, if a commodity costs twice as much in 1970 as it did in 1960, its index number would be 200 relative to 1960. Index numbers are used especially to compare business activity, the cost of living, and employment. They enable economists to reduce unwieldy business data into easily understood terms.

Index numbers generally are time series summarising movements in a group of related variables. In some cases, however, index numbers may compare geographic areas at a point in time. An example is a country's purchasing power parity. The best-known index number is the consumer price index, which measures changes in retail prices paid by consumers. In addition, a cost-of-living index (COLI) is a price index number that measures relative cost of living over time.

In contrast to a COLI based on the true but unknown utility function, a superlative index number is an index number that can be calculated. Thus, superlative index numbers are used to provide a fairly close approximation to the underlying cost-of-living index number in a wide range of circumstances.

There is a substantial body of economic analysis concerning the construction of index numbers, desirable properties of index numbers and the relationship between index numbers and economic theory.

Index number is a device, which is designed to measure changes in a variable or group of related variables with respect to time, geographic location or other characteristic, which cannot be measures directly. Index number may be simple or composite.

Simple index number is computed to measure a relative change in single variable with respect to a base. Composite index number is computed to measure average change in a group of related variables with respect to base. In economics and finance, an **index** is a statistical measure of changes in a representative group of individual data points. These data may be

derived from any number of sources, including company performance, prices, productivity, and employment.

Some indices display market variations that cannot be captured in other ways. For example, the Economist provides a Big Mac Index that expresses the adjusted cost of a globally ubiquitous Big Mac as a percentage over or under the cost of a Big Mac in the U.S. with a U.S. dollar (estimated: $3.57).

Norway prices reflect most relatively expensive Big Mac, at an 84% increase over U.S. prices, or $6.5725 U.S. The least relatively expensive Big Mac price occurs in Hong Kong, at a 52% reduction from U.S. prices, or $1.71 U.S. The Big Mac index is used to predict currency values. From this example, it would be assumed that Hong Kong currency is undervalued, and provides a currency investment opportunity.

Index Number Problem

The "Index Number Problem" refers to the difficulty of constructing a valid index when both price and quantity change over time. For instance, in the construction of price indices for inflation, the nature of goods in the economy changes over time as well as their prices. A price index constructed in 1950 using a standard basket of goods based on 1950 consumption would not well represent the prices faced by consumers in 2000, as goods in some categories are no longer traded in 2000, new categories of goods have been introduced, and the relative spending on different categories of goods will change drastically. Furthermore, the goods in the basket may have changed in quality.

There is no theoretically ideal solution to this problem. In practice for retail price indices, the "basket of goods" is updated incrementally every few years to reflect changes. Nevertheless, the fact remains that many economic indices taken over the long term are not really like-for-like comparisons and this is an issue taken into account by researchers in economic history

7.2   Purpose of Index Numbers

Following are the main uses:

A Barometer

Index numbers serve as a barometer for measuring the value of money.

With the help of index number we can easily make a comparison in the value

of money in different years.

Importance for the Government

The change in the value of money has a direct effect on the life of the public. So the Govt. adopts the fiscal and monetary policy according the results of index numbers.

Throws **Light on Economic Condition**

Index numbers are very helpful in comparing the economic conditions of a particular group at two different periods.

Consumption Standard

If we want to know the true consumption standard of a class in a locality we can compute the consumption index number.

Fixation of Wages

The money wages can be revised according the proportionate charge in the cost of living. The cost of living index number guides the Govt. and the executives for the fixation and revision of wages.

Importance for the producer

Price Index Number indicates the producer that he should expand the production or he should reduce the production. If price level is rising it means profit rate is high.

Analysis of Industry

If we want to judge the prospects of manufacturing concerns the investment index number can be constructed, to know the net yield of the industrial sector

To measure the average change in retail or wholesale prices or commonly or a group of commodities. To measure change in volume or quantity of goods produced or goods imported or exported. Index numbers are used to measure changes in a cost of living or total industrial production. The cost of living index numbers or consumer price index number is used to measure the effect of rise of fall prices on the general standard of living. Index numbers are used for forecasting. Index numbers are used to measure changes in qualitative expression like intelligence. Index number is also used to evaluate the purchasing power of money.

7.3   Paasche and Laspeyres model

The two most basic formulae used to calculate price indices are the **Paasche index** (after the German economist Hermann Paasche and the **Laspeyres index** (after the German economist Etienne Laspeyres

The **Paasche index** is computed as

$$P_P = \frac{\sum(p_{c,t_n} \cdot q_{c,t_n})}{\sum(p_{c,t_0} \cdot q_{c,t_n})}$$

while the **Laspeyres index** is computed as

$$P_L = \frac{\sum(p_{c,t_n} \cdot q_{c,t_0})}{\sum(p_{c,t_0} \cdot q_{c,t_0})}$$

where P is the change in price level, t_0 is the base period (usually the first year), and t_n the period for which the index is computed.

Note that the only difference in the formulas is that the former uses period n quantities, whereas the latter uses base period (period 0) quantities.

When applied to bundles of individual consumers, a Laspeyres index of 1 would state that an agent in the current period can afford to buy the same bundle as he consumed in the previous period, given that income has not changed; a Paasche index of 1 would state that an agent could have consumed the same bundle in the base period as she is consuming in the current period, given that income has not changed.

Hence, one may think of the Laspeyres index as one where the numeraire is the bundle of goods using base year prices but current quantities. Similarly, the Paasche index can be thought of as a price index taking the bundle of goods using current prices and current quantities as the numeraire.

The Laspeyres index tends to overstate inflation (in a cost of living framework), while the Paasche index tends to understate it, because the indices do not account for the fact that consumers typically react to price changes by changing the quantities that they buy. For example, if prices go up for good c then, *ceteris paribus*, quantities of that good should go down.

Fisher index and Marshall–Edgeworth index

A third index, the **Marshall–Edgeworth index** (named for economists Alfred Marshall and Francis Ysidro Edgeworth), tries to overcome these problems of under- and overstatement by using the arithmetic means of the quantities:

$$P_{ME} = \frac{\sum[p_{c,t_n} \cdot \frac{1}{2} \cdot (q_{c,t_0} + q_{c,t_n})]}{\sum[p_{c,t_0} \cdot \frac{1}{2} \cdot (q_{c,t_0} + q_{c,t_n})]} = \frac{\sum[p_{c,t_n} \cdot (q_{c,t_0} + q_{c,t_n})]}{\sum[p_{c,t_0} \cdot (q_{c,t_0} + q_{c,t_n})]}$$

A fourth, the **Fisher index** (after the American economist Irving Fisher), is calculated as the geometric mean of P_P and P_L:

$$P_F = \sqrt{P_P \cdot P_L}$$

Fisher's index is also known as the "ideal" price index.

However, there is no guarantee with either the Marshall–Edgeworth index or the Fisher index that the overstatement and understatement will thus exactly one cancel the other.

While these indices were introduced to provide overall measurement of relative prices, there is ultimately no way of measuring the imperfections of any of these indices (Paasche, Laspeyres, Fisher, or Marshall–Edgeworth) against reality.

Practical measurement considerations
 This section is empty. You can help by adding to it.

Normalizing index numbers

Price indices are represented as index numbers, number values that indicate relative change but not absolute values (i.e. one price index value can be compared to another or a base, but the number alone has no meaning). Price indices generally select a base year and make that index value equal to 100. You then express every other year as a percentage of that base year. In our example above, let's take 2000 as our base year. The value of our index will be 100. The price

- 2000: original index value was $2.50; $2.50/$2.50 = 100%, so our new index value is 100
- 2001: original index value was $2.60; $2.60/$2.50 = 104%, so our new index value is 104
- 2002: original index value was $2.70; $2.70/$2.50 = 108%, so our new index value is 108
- 2003: original index value was $2.80; $2.80/$2.50 = 112%, so our new index value is 112

When an index has been normalized in this manner, the meaning of the number 108, for instance, is that the total cost for the basket of goods is 4% more in 2001, 8% more in 2002 and 12% more in 2003 than in the base year (in this case, year 2000).

Relative ease of calculating the Laspeyres index

As can be seen from the definitions above, if one already has price and quantity data (or, alternatively, price and expenditure data) for the base period, then calculating the Laspeyres index for a new period requires only new price data. In contrast, calculating many other indices (e.g., the Paasche index) for a new period requires both new price data and new quantity data (or, alternatively, both new price data and new expenditure data) for each new period. Collecting only new price data is often easier than collecting both new price data and new quantity data, so calculating the Laspeyres index for a new period tends to require less time and effort than calculating these other indices for a new period.

Calculating indices from expenditure data

Sometimes, especially for aggregate data, expenditure data is more readily available than quantity data. For these cases, we can formulate the indices in terms of relative prices and base year expenditures, rather than quantities.

Here is a reformulation for the Laspeyres index:

Let E_{c,t_0} be the total expenditure on good c in the base period, then (by definition) we have $E_{c,t_0} = p_{c,t_0} \cdot q_{c,t_0}$ and therefore also $\frac{E_{c,t_0}}{p_{c,t_0}} = q_{c,t_0}$. We can substitute these values into our Laspeyres formula as follows:

$$P_L = \frac{\sum(p_{c,t_n} \cdot q_{c,t_0})}{\sum(p_{c,t_0} \cdot q_{c,t_0})} = \frac{\sum(p_{c,t_n} \cdot \frac{E_{c,t_0}}{p_{c,t_0}})}{\sum E_{c,t_0}} = \frac{\sum(\frac{p_{c,t_n}}{p_{c,t_0}} \cdot E_{c,t_0})}{\sum E_{c,t_0}}$$

A similar transformation can be made for any index.

Chained vs non-chained calculations

So far, in our discussion, we have always had our price indices relative to some fixed base period. An alternative is to take the base period for each time period to be the immediately preceding time period. This can be done with any of the above indices, but here's an example with the Laspeyres index, where t_n is the period for which we wish to calculate the index and t_0 is a reference period that anchors the value of the series:

$$P_{t_n} = \frac{\sum(p_{c,t_1} \cdot q_{c,t_0})}{\sum(p_{c,t_0} \cdot q_{c,t_0})} \times \frac{\sum(p_{c,t_2} \cdot q_{c,t_1})}{\sum(p_{c,t_1} \cdot q_{c,t_1})} \times \cdots \times \frac{\sum(p_{c,t_n} \cdot q_{c,t_{n-1}})}{\sum(p_{c,t_{n-1}} \cdot q_{c,t_{n-1}})}$$

Each term

$$\frac{\sum(p_{c,t_n} \cdot q_{c,t_{n-1}})}{\sum(p_{c,t_{n-1}} \cdot q_{c,t_{n-1}})}$$

answers the question "by what factor have prices increased between period t_{n-1} and period t_n". When you multiply these all together, you get the answer to the question "by what factor have prices increased since period t_0".

Nonetheless, note that, when chain indices are in use, the numbers cannot be said to be "in period t_0" prices.

Index number theory

Price index formulas can be evaluated based on their relation to economic concepts (like cost of living) or on their mathematical properties. Several different tests of such properties have been proposed in index number

theory literature. W.E. Diewert summarized past research in a list of nine such tests for a price index $I(P_{t0}, P_{tm}, Q_{t0}, Q_{tm})$, where P_0 and P_n are vectors giving prices for a base period and a reference period while Q_{t0} and Q_{tm} give quantities for these periods.

1. Identity test:

   $$I(p_{tm}, p_{tm}, \alpha \cdot q_{tm}, \beta \cdot q_{tn}) = 1 \quad \forall (\alpha, \beta) \in (0, \infty)^2$$
   The identity test basically means that if prices remain the same and quantities remain in the same proportion to each other (each quantity of an item is multiplied by the same factor of either α, for the first period, or β, for the later period) then the index value will be one.

2. Proportionality test:

   $$I(p_{tm}, \alpha \cdot p_{tm}, q_{tm}, q_{tn}) = \alpha \cdot I(p_{tm}, p_{tn}, q_{tm}, q_{tn})$$
   If each price in the original period increases by a factor α then the index should increase by the factor α.

3. Invariance to changes in scale test:

   $$I(\alpha \cdot p_{tm}, \alpha \cdot p_{tn}, \beta \cdot q_{tm}, \gamma \cdot q_{tn}) = I(p_{tm}, p_{tn}, q_{tm}, q_{tn}) \quad \forall (\alpha, \beta, \gamma) \in (0, \infty)^3$$
   The price index should not change if the prices in both periods are increased by a factor and the quantities in both periods are increased by another factor. In other words, the magnitude of the values of quantities and prices should not affect the price index.

4. Commensurability test:

   The index should not be affected by the choice of units used to measure prices and quantities.

5. Symmetric treatment of time (or, in parity measures, symmetric treatment of place):

   $$I(p_{tn}, p_{tm}, q_{tn}, q_{tm}) = \frac{1}{I(p_{tm}, p_{tn}, q_{tm}, q_{tn})}$$
   Reversing the order of the time periods should produce a reciprocal index value. If the index is calculated from the most recent time period to the earlier time period, it should be the reciprocal of the index found going from the earlier period to the more recent.

6. Symmetric treatment of commodities:

   All commodities should have a symmetric effect on the index. Different permutations of the same set of vectors should not change the index.

7. Monotonicity test:

$$I(p_{t_m}, p_{t_n}, q_{t_m}, q_{t_n}) \leq I(p_{t_m}, p_{t_r}, q_{t_m}, q_{t_r}) \quad \Leftarrow \quad p_{t_n} \leq p_{t_r}$$

   A price index for lower later prices should be lower than a price index with higher later period prices.

8. Mean value test:

   The overall price relative implied by the price index should be between the smallest and largest price relatives for all commodities.

9. Circularity test:

$$I(p_{t_m}, p_{t_n}, q_{t_m}, q_{t_n}) \cdot I(p_{t_n}, p_{t_r}, q_{t_n}, q_{t_r}) = I(p_{t_m}, p_{t_r}, q_{t_m}, q_{t_r}) \quad \Leftarrow \quad t_m \leq t_n \leq t_r$$

   Given three ordered periods t_m, t_n, t_r, the price index for periods t_m and t_n times the price index for periods t_n and t_r should be equivalent to the price index for periods t_m and t_r.

7.4 Limitations of index numbers

Review Questions

1. Discuss the advantages and disadvantages of index numbers
2. What is index numbers
3. Distinguish between Paasche and Laspeyres model

Unit 8
Breakeven analysis
8.1 Introduction

Break Even Analysis This is frequently used in big and economics to analyze the implication of various pricing and production decisions usually firms find themselves producing more than one product, these product lines may have a complimentary demand such that even though one product line is not profitable, its continued production is necessary to enhance demand for other product lines. In such instances, break-even analysis is important in defining economic quantities that may be produced.

Break-even analysis attempts to define different linear relationships, in respect to variable cost, total costs, total revenue and fixed costs.

By definition, Break-even analysis is a point at which the firm is neither making profits nor making losses.

Fixed costs; are those costs that remain constant at all levels of production over given period of time for example depreciation

$$\text{Depreciation} = \frac{\text{cost} - \text{swage value}}{\text{Useful life}}$$

Cost = 100, 000
S.V = 2000   = Dep. $\frac{100{,}000 - 20000}{}$
To last for 5 years

$$= 16,000$$

5 years
1rst 100000 – 16000 = 84000
2nd 84000 – 16000 = 68000
3rd 68000   - 16000 = 52000
4th 52000   - 16000 = 36000
5th 36000 – 16000   = 20000

B.E analysis assumes that such costs are constants but in the short run but the weakness is that the costs are never constraint

Variable costs; are those costs, which vary with the level of out put. They include; salaries, profit and loss expenses, raw materials, promotional costs, indirect costs etc.

B.E assumes that the variable cost per unit is constant

Total costs. It is the sum of fixed costs plus variable costs

Total revenue. Income realized from selling given number of units at a given price P hence R = PX Q = PQ

8.2   Contribution Ratio

It is given by the different between the selling Price and variable cost per unit. Contribution ratio = SP – VC@

Graphical Representation of FC, VC, TC, TR
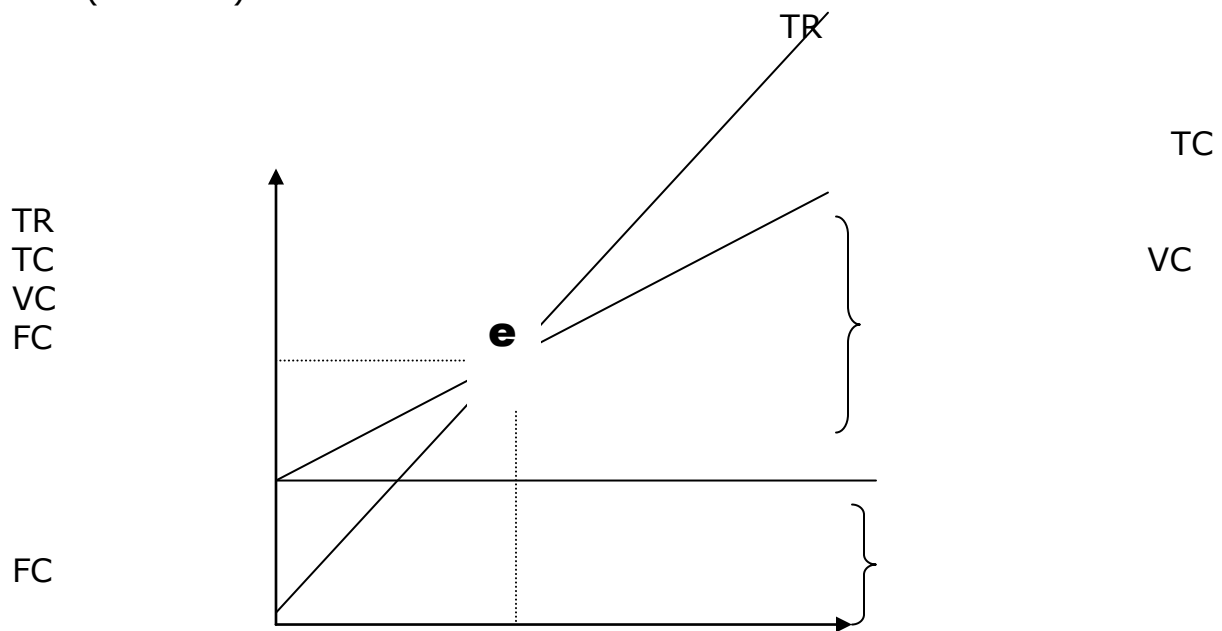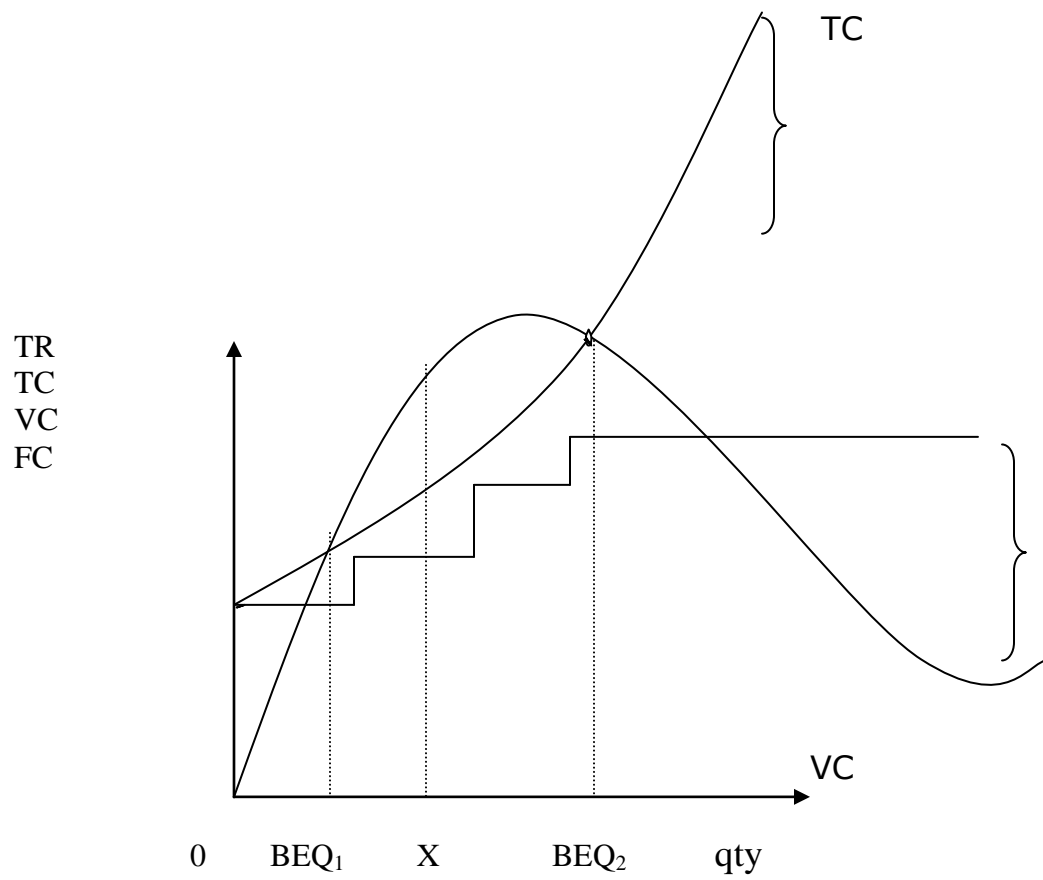**(Short run analysis)**

TR = P X Q
TC = FC X VC

T.R
T.C
Losses

V.C (Go increasing)

TR
TC
VC
FC

C   Profits

F.C are constant
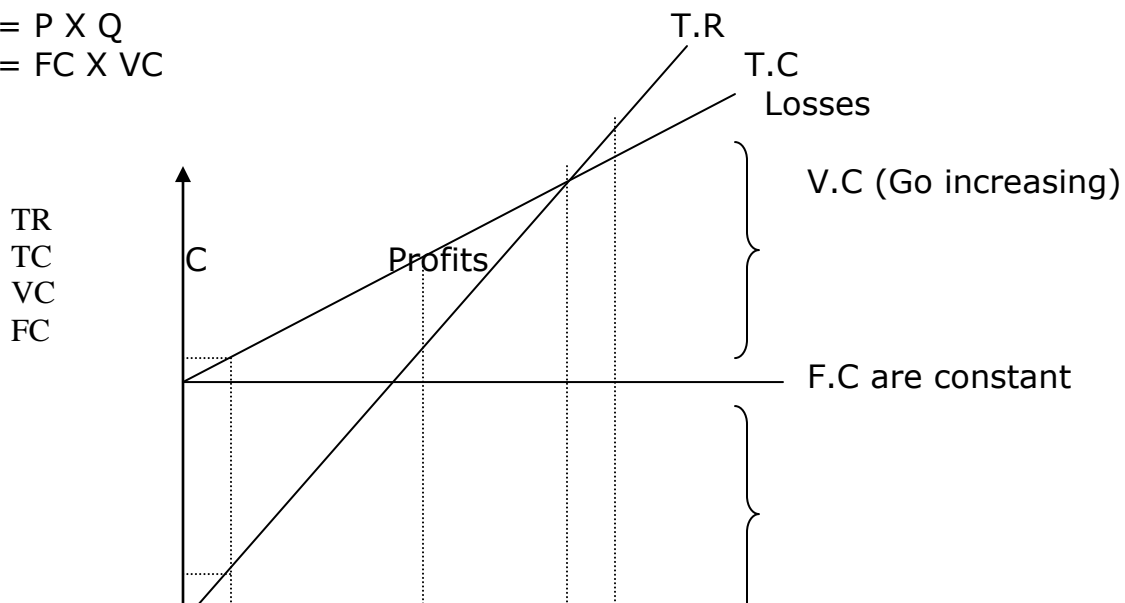
R
0

B.E point is a point (condition) where T.R is exactly equal to T.C (TR = TC)
BEP (TR = TC)

TR

TR
TC
VC
FC

TC

VC

e

FC

Long run analysis of the Break-even point

TC

TR
TC
VC
FC

VC

0   BEQ$_1$ X   BEQ$_2$ qty

FC

A V-shape

TR

Fixed costs are varying in the long run. Valuable costs take a V-shape
If the firm reaches point x, it will start making losses.

8.3   Weaknesses (Points Of Caution) Of Break-Even Analysis

1. It assumes that costs are constant for example fixed costs
2. It assumes that there are no closer costs like opportunity costs and utilization of the economies of scale for example construction industry; it should be constructed where there isn't any construction (it does not follow the rules of positive economics)
3. Linearity of curves. It assumes that the best assumption that there is a linear relationship between costs and revenue
4. Adequacy and validity of data. It yields inadequate data for instance the variable costs per Coco cola crate i.e VC @, VC@ = 24, (VC@ * 24Q) = 2400
5. Indiscriminate treatment of Factors of production: It lumps together too many products, too many plans from a break even chat for example MTN, Coco cola have many product lines and they want to see the B.E point
6. Planning activity. It assumes fixed costs are constant, which is not always the case.
7. Dynamic environment. It assumes that the environment is static (does not change)
8. Availability of market opportunities It assumes that whatever is produced is sold
9. It does not take into consideration other matters like social, political responsibility e.g. government intervention or giving back to society.

Derivation Of Break-Even Points

At BEP: TC = TR………………………….i
But   TC = FC + VC…………………ii
                 VC = VC @ Q………………….iii
Substituting equation (iii) into (ii)
                 TC = FC + VC @ Q…………….iv
Also   TR = PX Q = PQ…………………v
At BEP, equation (iv) = (v)
                 FC + VC@ Q = PQ
Make Q the subject
           FC = PQ – VC @
           FC = Q(P – VC @)

$$\frac{FC}{P-VC@} = \frac{Q(P - VC@)}{P-VC@}$$

$$Q = \frac{FC}{P - VC@}$$

Components of the Break-even quantity

Where Q = BE Qty
FC = Fixed cost
VC @ = VC Per unit
P = selling price

<u>Formula II for deriving B.E.P (including profits)</u>

Profits (Π) = TR – TC.............1
TR = PQ.............…..2
TC = FC + VC @ Q.............3
VC = VC @ Q.............…..4
Substitute equations 2, 3, and 4 in equation 1
Π = PQ – (FC + VC @ Q)
Π = PQ – FC – VC @ Q
Collect like terms
Π + FC = PQ – VC @ Q
Π + FC = Q(P – VC @)
Profit before tax
$Q = \dfrac{\Pi + FC}{P - VC\ @}$

Where: Π = profits
         P –VC @ is the contribution factor.

Example 2
Mukwano Industries whose reserve Price is 10 Shs are willing to supply five extra boxes per unit increase in price, at a price of 20 Shs. Per box. 50 boxes are bought and the rate at which the demand for soap varies with price change is 500 percent

3
Mukwano's variable cost per box is 5 units and makes a profit of 600 Shs when the market clears.
Required
Mukwano's fixed costs
As a manager for Mukwano industries if you were to make such a decision how consciously would you go about it basing on above scenario

Approach
What quantity should I produce to earn a profit?
Profit = TR – TC............. i
TR = PQ.............…..iii
Substitute equation iii & ii in i
   Π = PQ – (FC + VC@ Q)
Remove brackets
Π = PQ – FC – VC @ Q

Collect like terms

$\Pi + FC = PQ - VC\ 2\ Q$

$\Pi + FC = PQ -= VC @ Q$

$\Pi + FC = (QCP - VC @)$

$Q = \dfrac{\Pi + FC}{P - VC@}$

Reserve Px – 10

Supply 5 extra units per unit increase in Px at 20 per unit

50 units are consumed

Rate at which qty demanded varies with Px is $\dfrac{500}{3}$%

Variable cost per unit 5

Makes profit of 600/= when market clears.

At BEP   TR = TC

Π = TR – TC……………….1

TR = PQ……………………………2

TC = FC + VC…………………..3

VC = VCQ……………………….4

TC = FC + VC @ Q………………….5

Substitute equation 5 & 2 into 1

Π = PQ- (FC + VC @Q)…………………..6

Π = PQ – FC – VC @ Q…………………7

 Collect like terms

Π + FC = PQ – VC@ Q ………………..8

Π + FC = Q(P – VC @)…………………9

$\dfrac{\Pi + FC}{(P - VCQ)} = \dfrac{Q(P - VC @)}{(P - VC\ @)}$

$\dfrac{\Pi + FC}{(P - VC\ @)} = Q$ this is the formula

Relationship between supply and demand functions
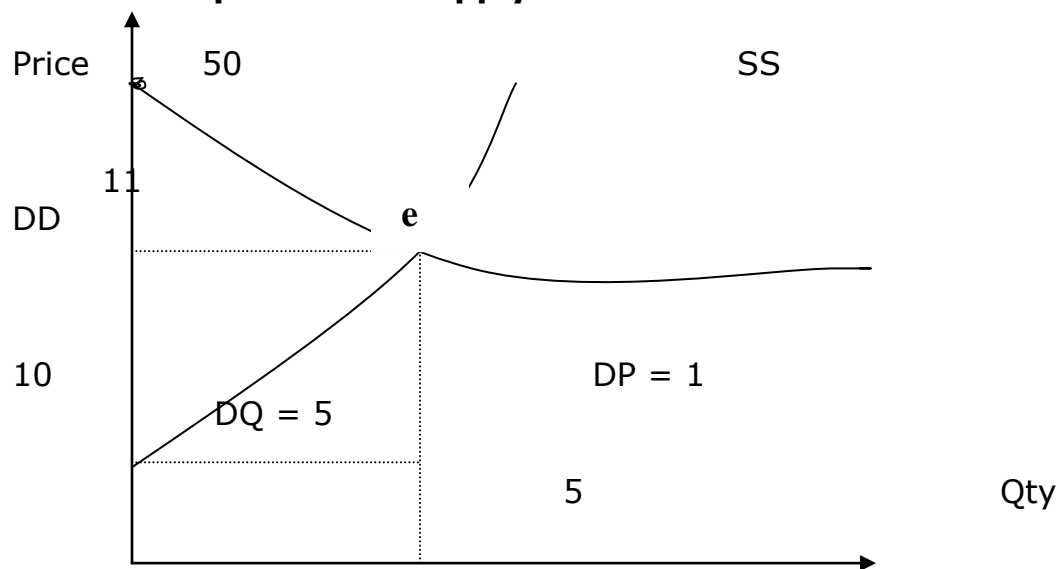
Supply functions is in the form of

P = MQ + C
P = Price
M = slope of lines (dd & ss)
Q = Qty
C = dd & ss intercept

Supply function slope

$M = \dfrac{DP}{DQ} = 1/5$ $\qquad$ $\dfrac{Ds}{Dd}$

P = 1/5 Q + C
But C = 10
 P = 1/5 Q + 10
Supply function is P = 1/5 Q + 10…………….i

Demand function slope

$M = \left(\dfrac{DP}{DQ} \right)$

Slope of DD curve = $\underline{M} $ = DQ

From the question $\dfrac{\dfrac{DP}{DQ}}{DP} = \dfrac{500\%}{3}$

$= \dfrac{500}{3} \times \dfrac{1}{100} = \dfrac{500}{300} = \dfrac{5}{3}$

$\dfrac{DQ}{DP} = \dfrac{DP}{DQ} = \dfrac{5}{3}$ X inverse

$\dfrac{DP}{DQ} = \dfrac{-3}{5}$

Using P = MQ + C

$P = \dfrac{-3}{5} Q + C$

But P = 20

Q = 50
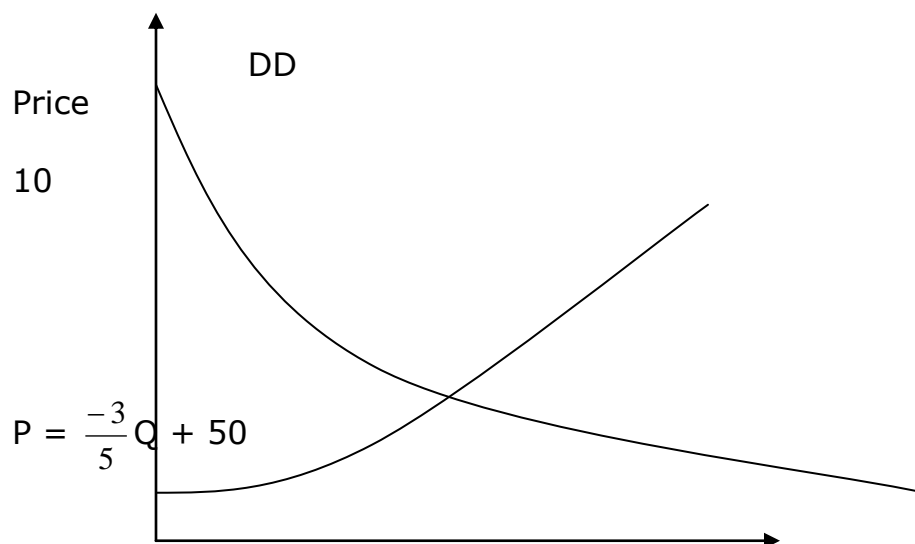
C = (Reserve price) (10)

$20 = \dfrac{-3}{5} \times C$

20 = -30 + C

20 + 30 = C

C = 50

Demand function = P = $\dfrac{-3}{5}$ Q + 50............(ii)

When the market clears

Price

10

P = $\dfrac{-3}{5}$ Q + 50

DD

P = 1/5 Q +

Qty

Supply function = Demand function

$$\frac{1}{5}Q + 10 = \frac{-3}{5}Q + 50$$

$$\frac{1}{5}Q + \frac{-3}{5}Q = 50 - 10$$

$$\frac{4}{5}Q = 40$$

$$Q = \frac{40 * 5}{4}$$

$$= \frac{200}{4} = 50$$

Q = 50

Substitute Q in any of the Equations to get P

$$P = \frac{1}{5} \times 50 + 10 \text{ (this one is for supply)}$$

P = 10 + 10

P = 20

Substitute the variables (P&Q) in the (BEQ) using the formula derived before

$$\frac{\Pi + FC}{(P - VCQ)} = Q$$

$$\frac{600 + FC}{20 - 5} = 50$$

$$\frac{600 + FC}{15} = 50$$

$$15 \times \frac{600 + FC}{15} = 50 \times 15$$

600 + FC = 750

FC = 750 - 600

FC = 150

Fixed costs = 150

Review Questions

4. Clearly explain how the fundamental economic question are addressed in a capitalist economic system
   (a) Define Economics,
   (b) Distinguish between;
   i) Microeconomics and Macroeconomics,
   ii) Positive Economics and Normative Economics.
5. Clearly state the fundamental questions that Economics addresses.
6. Distinguish between scarcity, choice and opportunity cost.
7. (a) What is a production possibility frontier (PPF)?
   (b) What factors may lead to a shift in a PPF?
8. Clearly explain the relationship between a PPF, scarcity, choice and opportunity cost.
9. What is the Economic implication of the following shapes of a PPF?
(i)   Convex PPF, Concave PPF, Linear but down ward sloping PPF

Review Questions

Philips the firm dealing in electronics but more particularly in Radios carries out the manufacturing and marketing activities of Radios. When the firm manufactures 2000.000 Radios at US $50 each, it realizes the total contribution of US $80,000,000 and earns US $24, 000, 000 after tax. The tax component is 25%

Required

a) Compute the revenue and cost functions
b) B.E quantity and income

Unit 9
Correlation and Regression
9.1   Introduction

Correlation is a measure of association between two variables. The variables are not designated as dependent or independent. The two most popular correlation coefficients are: Spearman's correlation coefficient rho and Pearson's product-moment correlation coefficient. When calculating a correlation coefficient for ordinal data, select Spearman's technique. For interval or ratio-type data, use Pearson's technique.

The value of a correlation coefficient can vary from minus one to plus one. A minus one indicates a perfect negative correlation, while a plus one indicates a perfect positive correlation. A correlation of zero means there is no relationship between the two variables. When there is a negative correlation between two variables, as the value of one variable increases, the value of the other variable decreases, and vise versa. In other words, for a negative correlation, the variables work opposite each other. When there is a positive correlation between two variables, as the value of one variable increases, the value of the other variable also increases. The variables move together.

The standard error of a correlation coefficient is used to determine the confidence intervals around a true correlation of zero. If your correlation coefficient falls outside of this range, then it is significantly different than zero. The standard error can be calculated for interval or ratio-type data (i.e., only for Pearson's product-moment correlation).

The significance (probability) of the correlation coefficient is determined from the t-statistic. The probability of the t-statistic indicates whether the observed correlation coefficient occurred by chance if the true correlation is zero. In other words, it asks if the correlation is significantly different than zero. When the t-statistic is calculated for Spearman's rank-difference correlation coefficient, there must be at least 30 cases before the t-distribution can be used to determine the probability. If there are fewer than 30 cases, you must refer to a special table to find the probability of the correlation coefficient.

Example
A company wanted to know if there is a significant relationship between the total number of salespeople and the total number of sales. They collect data for five months.

Variable 1	Variable 2

207	6907
180	5991
220	6810
205	6553
190	6190

Correlation coefficient = .921
Standard error of the coefficient = ..068
t-test for the significance of the coefficient = 4.100
Degrees of freedom = 3
Two-tailed probability = .0263

Another Example
Respondents to a survey were asked to judge the quality of a product on a four-point Likert scale (excellent, good, fair, poor). They were also asked to judge the reputation of the company that made the product on a three-point scale (good, fair, poor). Is there a significant relationship between respondents perceptions of the company and their perceptions of quality of the product?

Since both variables are ordinal, Spearman's method is chosen. The first variable is the rating for the quality the product. Responses are coded as 4=excellent, 3=good, 2=fair, and 1=poor. The second variable is the perceived reputation of the company and is coded 3=good, 2=fair, and 1=poor.

Variable 1	Variable 2
4	3
2	2
1	2
3	3
4	3
1	1
2	1

Correlation coefficient rho = .830

t-test for the significance of the coefficient = 3.332
Number of data pairs = 7
Probability must be determined from a table because of the small sample size.

9.2 Regression

Simple regression is used to examine the relationship between one dependent and one independent variable. After performing an analysis, the regression statistics can be used to predict the dependent variable when the independent variable is known. Regression goes beyond correlation by adding prediction capabilities.

People use regression on an intuitive level every day. In business, a well-dressed man is thought to be financially successful. A mother knows that more sugar in her children's diet results in higher energy levels. The ease of waking up in the morning often depends on how late you went to bed the night before. Quantitative regression adds precision by developing a mathematical formula that can be used for predictive purposes.

For example, a medical researcher might want to use body weight (independent variable) to predict the most appropriate dose for a new drug (dependent variable). The purpose of running the regression is to find a formula that fits the relationship between the two variables. Then you can use that formula to predict values for the dependent variable when only the independent variable is known. A doctor could prescribe the proper dose based on a person's body weight.

The regression line (known as the *least squares line*) is a plot of the expected value of the dependent variable for all values of the independent variable. Technically, it is the line that "minimizes the squared residuals". The regression line is the one that best fits the data on a scatterplot.

Using the regression equation, the dependent variable may be predicted from the independent variable. The slope of the regression line (b) is defined as the rise divided by the run. The y intercept (a) is the point on the y axis where the regression line would intercept the y axis. The slope and y intercept are incorporated into the regression equation. The intercept is usually called the constant, and the slope is referred to as the coefficient. Since the regression model is usually not a perfect predictor, there is also an error term in the equation.

In the regression equation, y is always the dependent variable and x is always the independent variable. Here are three equivalent ways to mathematically describe a linear regression model.
y = intercept + (slope ±x) + error
y = constant + (coefficient±x) + error
y = a + bx + e

The significance of the slope of the regression line is determined from the t-statistic. It is the probability that the observed correlation coefficient occurred by chance if the true correlation is zero. Some researchers prefer to report the F-ratio instead of the t-statistic. The F-ratio is equal to the t-statistic squared.

The t-statistic for the significance of the slope is essentially a test to determine if the regression model (equation) is usable. If the slope is significantly different than zero, then we can use the regression model to predict the dependent variable for any value of the independent variable.

On the other hand, take an example where the slope is zero. It has no prediction ability because for every value of the independent variable, the prediction for the dependent variable would be the same. Knowing the value of the independent variable would not improve our ability to predict the dependent variable. Thus, if the slope is not significantly different than zero, don't use the model to make predictions.

The coefficient of determination (r-squared) is the square of the correlation coefficient. Its value may vary from zero to one. It has the advantage over the correlation coefficient in that it may be interpreted directly as the proportion of variance in the dependent variable that can be accounted for by the regression equation. For example, an r-squared value of .49 means that 49% of the variance in the dependent variable can be explained by the regression equation. The other 51% is unexplained.
The standard error of the estimate for regression measures the amount of variability in the points around the regression line. It is the standard deviation of the data points as they are distributed around the regression line. The standard error of the estimate can be used to develop confidence intervals around a prediction.

Example
A company wants to know if there is a significant relationship between its advertising expenditures and its sales volume. The independent variable is advertising budget and the dependent variable is sales volume. A lag time of one month will be used because sales are expected to lag behind actual advertising expenditures.

Data was collected for a six month period. All figures are in thousands of dollars. Is there a significant relationship between advertising budget and sales volume?

Indep. Var.	Depen. Var
4.2	27.1
6.1	30.4
3.9	25.0
5.7	29.7
7.3	40.1
5.9	28.8

Model: y = 9.873 + (3.682±x) + error
Standard error of the estimate = 2.637
t-test for the significance of the slope = 3.961
Degrees of freedom = 4
Two-tailed probability = .0149
r-squared = .807

You might make a statement in a report like this: A simple linear regression was performed on six months of data to determine if there was a significant relationship between advertising expenditures and sales volume. The t-statistic for the slope was significant at the .05 critical alpha level, t(4)=3.96, p=.015. Thus, we reject the null hypothesis and conclude that there was a positive significant relationship between advertising expenditures and sales volume. Furthermore, 80.7% of the variability in sales volume could be explained by advertising expenditures.

9.3   Mathematical Presentations of Correlation

For random variables $X(s)$ and $X(t)$ at different points s and t of some space, the correlation function is
$$C(s,t) = \text{corr}(X(s), X(t)),$$ where corr is described in the article on correlation. In this definition, it has been assumed that the stochastic variable is scalar-valued. If it is not, then more complicated correlation functions can be defined. For example, if one has a vector $X_i(s)$, then one can define the matrix of correlation functions

$$C_{ij}(s, s') = \mathrm{corr}(X_i(s), X_j(s'))$$ or a scalar, which is the trace of this matrix. If the probability distribution has any target space symmetries, i.e. symmetries in the space of the stochastic variable (also called **internal symmetries**), then the correlation matrix will have induced symmetries. If there are symmetries of the space (or time) in which the random variables exist (also called **spacetime symmetries**) then the correlation matrix will have special properties. Examples of important spacetime symmetries are ;-

- **translational symmetry** yields $C(s,s') = C(s - s')$ where s and s' are to be interpreted as vectors giving coordinates of the points
- **Rotational symmetry** in addition to the above gives $C(s, s') = C(|s - s'|)$ where $|x|$ denotes the norm of the vector x (for actual rotations this is the Euclidean or 2-norm).

n is

$$C_{i_1 i_2 \cdots i_n}(s_1, s_2, \cdots, s_n) = \langle X_{i_1}(s_1) X_{i_2}(s_2) \cdots X_{i_n}(s_n) \rangle.$$

If the random variable has only one component, then the indices i_j are redundant. If there are symmetries, then the correlation function can be broken up into irreducible representations of the symmetries — both internal and spacetime.

The case of correlations of a single random variable can be thought of as a special case of autocorrelation of a stochastic process on a space which contains a single point.

Properties of probability distributions

With these definitions, the study of correlation functions is equivalent to the study of probability distributions. Probability distributions defined on a finite number of points can always be normalized, but when these are defined over continuous spaces, then extra care is called for. The study of such distributions started with the study of random walks and led to the notion of the Ito calculus.

The Feynman path integral in Euclidean space generalizes this to other problems of interest to statistical mechanics. Any probability distribution which obeys a condition on correlation functions called reflection positivity lead to a local quantum field theory after Wick rotation to Minkowski spacetime. The operation of renormalization is a specified set of mappings from the space of probability distributions to itself. A quantum field theory is called renormalizable if this mapping has a fixed point which gives a quantum field theory.

1.   Discuss the Mathematical Presentations of Correlation

2.   Distinguish between Correlation and Regression

Unit 10:   Basic calculus

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