KAMPALA INTERNATIONAL UNIVERSITY

PHY 1203 INTRODUCTORY SOLID STATE PHYSICS

STUDY UNIT 3: Mechanical Properties of Solids

Mechanical Properties of Solids

• Inter molecular Force

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as inter molecular forces.

• Elasticity

The property of the body to regain its original configuration (length, volume or shape) when the deforming forces are removed, is called elasticity.

• The change in the shape or size of a body when external forces act on it is determined by the forces between its atoms or molecules. These short range atomic forces are called elastic forces.

• Perfectly elastic body

A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body. Quartz and phospher bronze are the examples of nearly perfectly elastic bodies.

• Plasticity

The inability of a body to return to its original size and shape even on removal of the deforming force is called plasticity and such a body is called a plastic body.

• Stress

Stress is defined as the ratio of the internal force F, produced when the substance is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In other words,

$$\text{Stress} = \frac{F}{A}$$

The SI unit of stress is newton per square metre (Nm^{-2}). In CGS units, stress is measured in dyne cm^{-2}. Dimensional formula of stress is $[ML^{-1}T^{-2}]$.

• Stress is of two types:

(i) Normal stress: It is defined as the restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types: tensile stress and compressive stress.

(ii) Tangential stress: When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.

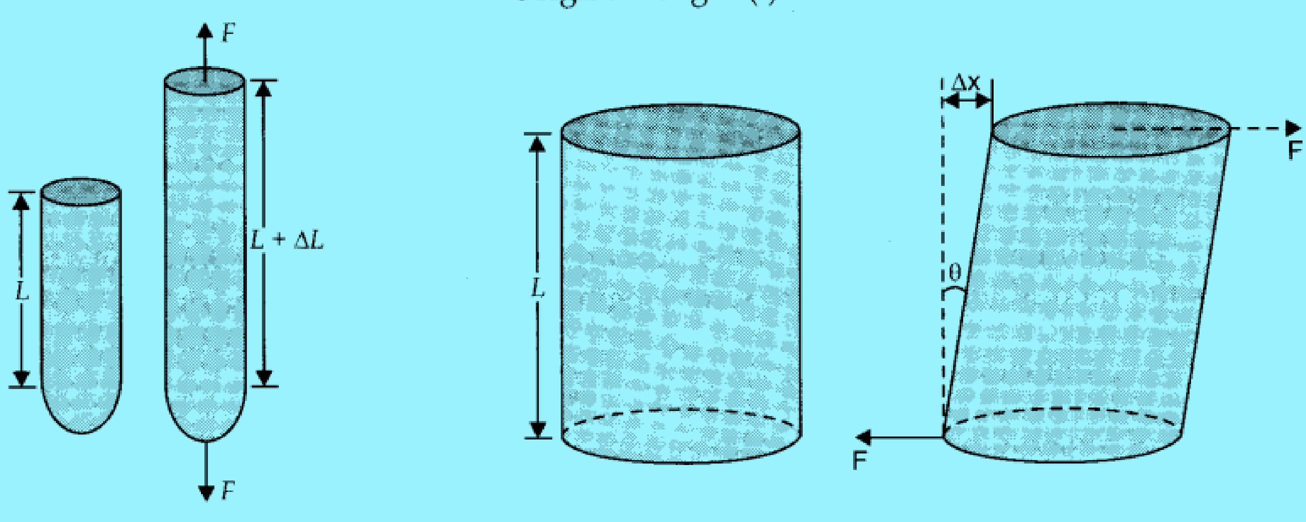
• Strain

It is defined as the ratio of the change in size or shape to the original size or shape. It has no dimensions, it is just a number.

Strain is of three types:

(i) Longitudinal strain: If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as:

$$\text{Longitudinal strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } (l)}$$



(a) (b)

(ii) Volumetric strain: If the deforming force produces a change in volume alone, the strain produced in the body is called volumetric strain. It is given as:

$$\text{Volumetric strain} = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

(iii) Shear strain: The angle tilt caused in the body due to tangential stress expressed is called shear strain. It is given as:

$$\text{Shear strain} = \theta = \frac{\Delta L}{L}.$$

• The maximum stress to which the body can regain its original status on the removal of the deforming force is called elastic limit.

• Hooke's Law

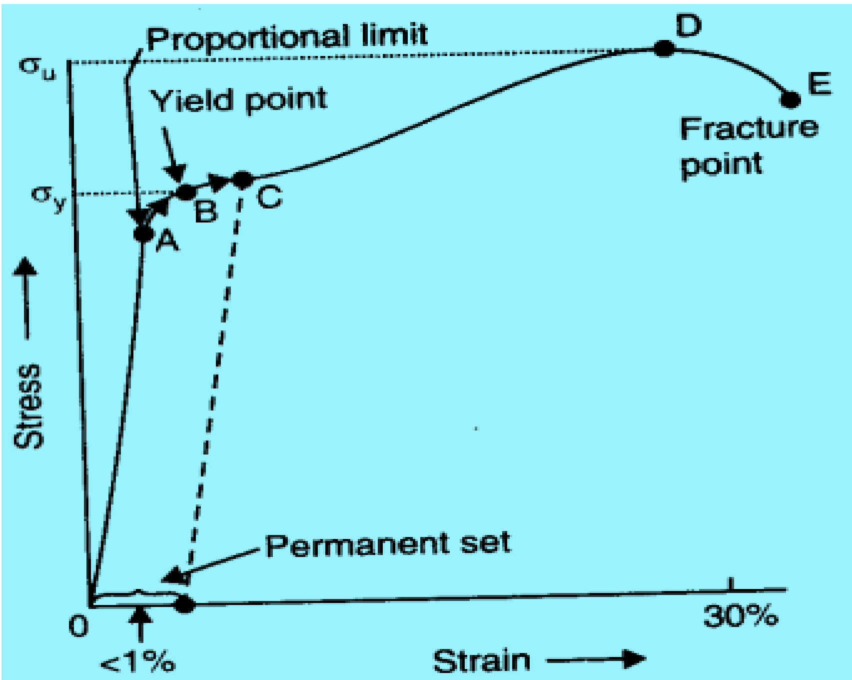
Hooke's law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity. Thus

$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

Since strain is a pure number, the units of this constant are the same as those of stress, *i.e.*, Nm^{-2}.

• Stress Strain Curve

Stress strain curves are useful to understand the tensile strength of a given material. The given figure shows a stress-strain curve of a given metal.



• The curve from O to A is linear. In this region Hooke's Proportional limit law is obeyed.

• In the region from A to 6 stress and strain are not . proportional. Still, the body regains its original dimension, once the load is removed.

• Point B in the curve is yield point or elastic limit and the corresponding stress is known as yield strength of the material.

• The curve beyond B shows the region of plastic deformation.

• The point D on the curve shows the tensile strength of the material. Beyond this point, additional strain leads to fracture, in the given material.

• Young's Modulus

For a solid, in the form of a wire or a thin rod, Young's modulus of elasticity within elastic limit is defined as the ratio of longitudinal stress to longitudinal strain. It is given as:

$$\text{Young's modulus, } Y = \frac{F/A}{\Delta l/l} = \frac{Fl}{A.\Delta l} = \frac{mgl}{\pi r^2.\Delta l}$$

It has the unit of longitudinal stress and dimensions of $[ML^{-1}T^{-2}]$. Its unit is Pascal or N/m^2.

• Bulk Modulus

Within elastic limit the bulk modulus is defined as the ratio of longitudinal stress and volumetric strain. It is given as:

$$\text{Bulk modulus, } B = \frac{F/A}{\Delta V/V} = -\frac{P}{\Delta V/V}$$

– ve indicates that the volume variation and pressure variation always negate each other.

• Reciprocal of bulk modulus is commonly referred to as the “compressibility”. It is defined as the fractional change in volume per unit change in pressure.

• Shear Modulus or Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain.

Modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}} = \frac{F/A}{\theta}.$$

• Poisson’s Ratio

The ratio of change in diameter (ΔD) to the original diameter (D) is called lateral strain. The ratio of change in length (Δl) to the original length (l) is called longitudinal strain. The ratio of lateral strain to the longitudinal strain is called Poisson’s ratio.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = -\frac{\Delta D/D}{\Delta l/l}$$

For most of the substances, the value of σ lies between 0.2 to 0.4. When a body is perfectly incompressible, the value of σ is maximum and equals to 0.5.

• Elastic Fatigue

It is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming forces.

• Relations between Elastic Moduli

For isotropic materials (i.e., materials having the same properties in all directions), only two of the three elastic constants are independent. For example, Young's modulus can be expressed in terms of the bulk and shear moduli.

$$\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3B}$$

Also, $Y = 3B(1 - \sigma) = 2\eta(1 + \sigma)$

• Breaking Stress

The ultimate tensile strength of a material is the stress required to break a wire or a rod by pulling on it. The breaking stress of the material is the maximum stress which a material can withstand. Beyond this point breakage occurs.

- When a wire of original length L is stretched by a length l by the application of force F at one end, then

Work done to stretch wire $= \frac{1}{2} \times$ stretching force $\times$ extension

$$= \frac{1}{2}\frac{YAl^2}{L}$$

- Work done per unit volume of wire is given as:

$$W = \frac{1}{2}\text{Stress} \times \text{strain.}$$

According to the formula given by

$$Y = \frac{F \cdot L}{Al}$$

Where F is the force needed to stretch the wire of length L and area of cross-section A. l is the increase in the length of the wire.

$$\therefore \qquad F = \frac{YAl}{L}$$

The work done by this force in stretching the wire is stored in the wire as potential energy.

$$\therefore \qquad dW = F \times dl$$

$$= \frac{YAl}{L} \cdot dl$$

Integrating both sides, we get

$$W = \frac{YA}{L}\int_0^l l \cdot dl$$

$$W = \frac{YA}{L}\left[\frac{1}{2}l^2\right]_0^l$$

$$= \frac{YA}{L}\left[\frac{1}{2}l^2\right]$$

$$= \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot l$$

$$= \frac{1}{2}\cdot F\cdot l$$

Which equal to the elastic potential energy U.

$$\therefore \quad U = \frac{1}{2}F\cdot l = \frac{1}{2}\times \text{Force}\times \text{extension}$$

Now the potential energy per unit volume is

$$\frac{1}{2}\frac{F\cdot l}{V} = \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot\frac{l}{V}$$

$$\Rightarrow \quad \frac{1}{2}\times\frac{Fl}{AL} = \frac{1}{2}\left(\frac{YAl}{L}\right)\cdot\frac{l}{AL} \qquad [V = AL]$$

$$= \frac{1}{2}\left(\frac{Yl}{L}\right)\cdot\frac{l}{L}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)\cdot\frac{l}{L}$$

$$= \frac{1}{2}\times \text{Stress}\times\text{Strain}$$

Hence, the elastic potential energy of a wire (energy density) is equal to half the product of its stress and strain.

• IMPORTANT TABLES

Table 3.1 Young's Moduli, elastic Limit and tensile strengths of some materials

Substance	*Young's modulus* $10^9 N/m^2$ σ_y	*Elastic limit* $10^7 N/m^2$ %	*Tensile strength* $10^7 N/m^2$ σ_u
Aluminium	70	18	20
Copper	120	20	40
Iron (wrought)	190	17	33
Steel	200	30	50
Bone			
(Tensile)	16		12
(Compressive)	9		12

Table 3.2 Stress, Strain and various elastic moduli

Type of Stress	Stress	Strain	Change in		Elastic modulus	Name of modulus	State of Mater
			shape	volume			
Tensile or Compressive	Two equal and opposite forces perpendicular to opposite faces ($\sigma = F/A$)	Elongation or compression parallel to force direction ($\Delta L/L$) (longitudinal strain)	Yes	No	$Y = (F \times L) / (A \times \Delta L)$	Young's modulus	Solid
Shearing	Two equal and opposite forces parallel to opposite surfaces [forces in each case such that total force and total torque on the body vani-shes ($\sigma_s = F/A$)]	Pure shear, θ	Yes	No	$G = (F \times \theta) / A$	Shear modulus	Solid
Hydraulic	Forces perpendicular every where to the surface, force per unit area (pressure) same everywhere	Volume change (compression or elongation) ($\Delta V/V$)	No	Yes	$B = -p/(\Delta V/V)$	Bulk modulus	Solid, liquid and gas

Point Defects and Dislocations

A perfect crystal, with every atom of the same type in the correct position does not exist. There always exist crystalline defects. Defects are more than small annoying perturbations of the perfect crystal. In the present context, they are essential for explaining the mechanical properties, and later we will see that they are also indispensable for phenomena like electrical resistance or for the electronic properties of semiconductors.

Any deviation from the perfect atomic arrangement in a crystal is said to contain a defect. A crystalline defect is a lattice irregularity having one or more of its dimensions on the order of an atomic dimension.

- Point Defects: Occur at a single lattice point; that is, an atom missing or is in an irregular place in the lattice structure.

- Linear Defects: Occur along a row of atoms.
- Planar defects: Occur over two dimensional surface in acrystal

Defects are crucial to the behavior of materials. Almost all technology involving materials depend on the existence of some kind of defects. Crystal imperfections influence properties of crystals such as strength, electrical conductivity and hysteresis loss of ferromagnetism. Thus some properties of crystals are controlled by as much as by imperfections and by the nature of the host crystals. For example;

- Conductivity of some semiconductors is entirely due to trace amount of chemical impurities.
- Color, luminescence of some crystals arises from impurities and imperfections.
- Atomic diffusion may be accelerated enormously by impurities or imperfections.

Vacancies

A vacancy is an empty space(empty lattice sites) where an atom should be, but is missing.

The higher the temperature, the more often atoms are jumping from one equilibrium position to another, and a larger number of vacancies can be found on the crystal. The equilibrium number of vacancies N_v increases exponentially with absolute temperature and can be estimated using Boltzmann distribution;

$$N_v = Nexp^{\left(\frac{-E_v}{kT}\right)}$$

Where: N is the number of regular lattice sites

K is Boltzmann constant

E_v is the energy needed to form a vacancy

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QUIZ

1) What typically happens when a crystal is exposed to a small stress?

2) How can an elastic deformation of a crystal be described microscopically, and why would you expect Hooke's law to hold for a small strain?

3) How do the stress/strain curves look for a typical ductile and brittle material?.

REFERENCES

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