

Study Unit 2: Electric Field Intensity

Introduction

This unit is continuous of electric force it mainly describes electric field intensity and gauss's law of electrostatics. A very useful concept is the electric field, which is defined as the force per unit charge. Every point in space has a unique electric field associated with it. We can define the flux of the electric field through a given surface. This leads us to Gauss's law, which is an alternative way of stating Coulomb's law. In cases involving sufficient symmetry, it is much quicker to calculate the electric field via Gauss's law than via Coulomb's law and direct integration. Finally, we discuss the energy density in the electric field, which provides another way of calculating the potential energy of a system.

Learning Outcomes of Study Unit 2

Upon completion of this study unit, you should be able to

- 2.1 Define electric field intensity and states its units
- 2.2 State the superposition as applied to electric field intensity
- **2.3** Explain the three types of continuous charge distribution (linear, surface and volume charge density)
- 2.4 Define electric field lines and state its properties
- 2.5 Sketch the electric field patterns of some common charge distribution
- 2.6 Define electric flux through a given surface
- 2.7 State and derive the Gauss's law in the most general form
- 2.8 Use Gauss's law to calculate electric field intensity due to:
 - (i) A charge conducting sphere
 - (ii) A line of charge
 - (iii) Uniform spherical distribution of charge
 - (iv) A uniformly cylindrical distribution of charge



2.1 Electric field intensity

The electric force is a field force. An electric field is said to exist in the region of space around a charged object. This charged object is the source charge. When another charged object, the test charge, enters this electric field, an electric force act on it.

The electric field vector, \vec{E} , at a point in space is defined as the electric force acting on a positive test charge q_0 , placed at that point divided by the test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_0} = k \frac{q}{r^2}$$

The SI units of \vec{E} are NC^{-1} .

2.1.1 Direction of electric field \vec{E}

Force on +q is with	Force on -q is against	Force on +q is with	Force on -q is against
the field direction	field direction	field direction	field direction
F = I = I = I = I = I = I = I = I = I =	-q E F F Q + Q +	+q Fr Fr	$F \rightarrow F \rightarrow F$

Example 1

A +2nc charge is placed at a distance r from $-8\mu c$ if the charge experiences a force 400N as shown below, what is electric field intensity at P





Solution

First we note that the direction of \vec{E} is towards +q

$$\vec{E} = \frac{\vec{F_e}}{q}$$
$$\vec{E} = \frac{400}{2x10^{-9}}$$
$$\vec{E} = 2x10^{12}N/C \text{ (Downwards)}$$

Example 2

An electron with a speed of $3.00 \times 10^6 m/s$ moves into a uniform electric field of 1000 N/C. The field is parallel to the electron's motion. How far does the electron travel before it is brought to rest?

Solution

When the electron, with mass $me = 9.11 \times 10^{-31} kg$, enters the electric field, it experiences a retarding force given by F = -eE

negative since the E-field slows the electron. Since this force slows the electron, it produces a deceleration. Using Newton's 2nd law we can write

$$a = \frac{F}{m_e}$$



$$a = \frac{-eE}{m_e}$$

$$a = \frac{-1.6x10^{-19}x1000}{9.11 \times 10^{-31}} = -1.7x10^{14}m/s^2$$

Using the 3rd equation of motion, we can find the distance that the electron travels before coming to a stop Δx

$$v^{2} = u^{2} + 2a\Delta x$$
$$\Delta x = \frac{v^{2} - u^{2}}{2a}$$
$$\Delta x = \frac{0 - (3.00 \times 10^{6} \text{ m/s})^{2}}{2(-1.7x10^{14} \text{ m/s}^{2})} = 0.0256m$$

2.2 Superposition principle and electric field intensity

The resultant field electric field (\vec{E}) in the vicinity of a number of point charges is equal to the vector sum of the fields due to each charge taken individually. Consider a number N of point charges Q_1 , Q_2 , Q_3 , and Q_n at distances r_1 , r_2 , r_3 and r_n from the test charge Q_t





Let \hat{r}_1 be a unit vector pointing away from point charge Q_i towards Q_t

The electric force on Q_t is

$$\vec{F}_E = k \frac{\sum_i^n Q_t Q_i}{{r_i}^2} \hat{r}_i$$

The force on 1C of test charge is $\vec{E} = \frac{\vec{F}_E}{Q_t} = k \sum \frac{Q_i}{r_i^2} \hat{r}_i$



Solution



Using

$$\vec{E} = k \frac{q}{r^2}$$
$$\vec{E}_A = \frac{9.0x10^9 x 6.0x10^{-9}}{(6.0x10^{-2})^2}$$
$$\vec{E}_A = 15000N/C$$

$$\vec{E}_C = \frac{9.0x10^9 x 8.0x10^{-9}}{(3.0x10^{-2})^2}$$
$$\vec{E}_C = 80000N/C$$



 $\vec{E} = 15000 + 80000$

$\vec{E} = 95000N/C$ (Towards right)

Example 2

Positive charges are situated at three corners of a rectangle as shown in the figure below. Find the electric field at the fourth corner



Solution





From the geometry of the rectangle as shown in the figure above, we have

$$r_{3} = \sqrt{(r_{1})^{2} + (r_{2})^{2}} = \sqrt{5.00^{2} + 2.00^{2}} = 5.4cm$$

And angle $\phi = tan^{-1} \left(\frac{r_{2}}{r_{1}}\right)$
 $\phi = tan^{-1} \left(\frac{5.00}{2.00}\right)$
 $\phi = 68.2^{0}$

The components of the individual E-field vectors are

Using
$$\vec{E} = k \frac{q}{r^2}$$

 $\vec{E}_1 = \frac{9.00x10^9x6.00x10^{-9}}{(5.00x10^{-2})^2} = 21600N/C$

$$\vec{E}_2 = \frac{9.00x10^9x4.00x10^{-9}}{(2.00x10^{-2})^2}$$
$$\vec{E}_2 = 90000N/C$$

$$\vec{E}_3 = \frac{9.00x10^9x7.00x10^{-9}}{(5.400x10^{-2})^2}$$
$$\vec{E}_3 = 21604.9N/C$$

Extract





Resolving vertically

$$\vec{E}_y = \vec{E}_2 + \vec{E}_3 sin\phi$$

 $\vec{E}_y = 90000 + 21604.9 sin62.8$
 $\vec{E}_y = 109215.8 N/C$

Resolving horizontally

$$\vec{E}_x = -\vec{E}_1 - \vec{E}_3 cos\phi$$

 $\vec{E}_x = -21600 - 21604.9 cos62.8$
 $\vec{E}_x = -31475.6 N/C$

Thus, resultant electric field is

$$\vec{E} = \sqrt{(\vec{E}_x)^2 + + (\vec{E}_y)^2}$$
$$\vec{E} = \sqrt{(31475.6)^2 + (109215.8)^2}$$
$$\vec{E} = 111307.2N/C$$
$$\theta = \tan^{-1} \left(\frac{\vec{E}_y}{\vec{E}_x}\right)$$
$$\theta = \tan^{-1} \left(\frac{109215.8}{-31475.6}\right)$$
$$\theta = -73.9^0$$

Direction

$$\theta = -73.9$$



However, this is with respect to the -x axis. θ (with respect to the +x axis) is then

$$\theta = 180 - 73.9$$

$$\theta = 106.1^{\circ}$$

2.3 Continuous distribution of charge

There are three types of continuous charge distribution

- Linear charge distribution
- Surface charge distribution
- Volume charge distribution

2.3.1 Linear charge distribution

If the charge is not evenly distributed over a length of a conductor, Then it is said to be linear charge distribution. It is referred as linear charge density (λCm^{-1})

Consider a conductor of length l with the surface load density of λ and an aspect of dl on it the small charge on it will be

$$dq = \lambda dl$$

The electric field on the small charge element dq at point P is



$$d\vec{E} = k\frac{dq}{r^2}\hat{r}$$



Where \hat{r} is the unit vector pointing from the element to the point P

$$d\vec{E}_p = k \frac{\lambda dl}{r^2} \hat{r}$$

Integrating both sides

$$\int d\vec{E}_p = \int_0^l k \frac{\lambda dl}{r^2} \hat{r}$$

$$\vec{E}_p = \frac{k}{r^2} \int_0^l \lambda dl \hat{r}$$

2.3.2 Surface charge distribution

When the charge is uniformly distributed over the conductor's surface, is called surface charge distribution or surface charge density (σCm^{-2}) it can also be defined as charge per unit area.

$$\sigma = \frac{dq}{ds}$$

Where dq is the small element of charge and ds a small surface

$$dq = \sigma ds$$





The electric field on the small charge element dq at point P is

$$d\vec{E}_p = k \frac{\sigma ds}{r^2} \hat{r}$$
 Where s is the surface

Integrating both sides

$$\int d\vec{E}_p = \int_0^s k \frac{\sigma ds}{r^2} \hat{r}$$
$$\vec{E}_p = \frac{k}{r^2} \int_0^s \sigma ds \hat{r}$$

2.3.3 Volume charge distribution

When the charge is distributed over the drivers volume, it is called volume charge distribution or volume charge density(ρCm^{-3}), it is also define as charge per unit volume.

The density of the volume charge is given by $\rho = \frac{dq}{d\tau}$ where τ is the volume element $dq = \rho d\tau$



The electric field on the small charge element dq at point P is $d\vec{E}_p = k \frac{\rho d\tau}{r^2} \hat{r}$

Integrating both sides

$$\int d\vec{E}_p = \int_0^\tau k \frac{\rho d\tau}{r^2} \hat{r}$$



 $\vec{E}_p = \frac{k}{r^2} \int_0^\tau \rho d\tau \hat{r}$



By symmetry, the components parallel to \hat{j} and \hat{k} cancel so the electric field is confined to the x direction (\hat{i}). The magnitude of the electric field at P due to the element λdl is

$$d\vec{E}_x = d\vec{E}\,\cos\theta$$
$$d\vec{E}_x = \left(\frac{x}{r}\right)k\frac{\lambda dl}{r^2}\hat{x}$$



$$d\vec{E}_x = \frac{kx\lambda dl}{r^3}\hat{x}$$

Integrating both sides

$$\vec{E}_x = k \int_0^{2\pi a} \frac{kx\lambda dl}{(x^2 + a^2)^{\frac{3}{2}}} \hat{x}$$
$$\vec{E}_x = \frac{kx\lambda(2\pi a)}{(x^2 + a^2)^{\frac{3}{2}}} \hat{x}$$
But $\lambda = \frac{Q}{2\pi a}$

Hence

$$\vec{E}_x = k \frac{Qx}{(x^2 + a^2)^{\frac{3}{2}}} \hat{x}$$

Where \hat{x} is the unit vector along x-direction

A graph of \vec{E}_x against x has the form shown below, When x = 0 $\vec{E}_x = 0$ When $x \gg a$ $\vec{E}_x = \frac{Q}{x^2}\hat{x}$





Example 2:

Electric field intensity at a point on the axis of a uniformly charged disc of radius R

Charge per unit are $\sigma = \frac{Q}{\pi R^2}$

If the charge density is σCm^{-2} , then amount of charge on the ring is $dQ = (2\pi r dr)\sigma$

Refer to the result in the previous example we can proceed and find the electric field intensity.

$$d\vec{E}_x = \frac{kxdQ}{(x^2 + r^2)^{\frac{3}{2}}}\hat{x}$$

Substituting for dQ we obtain

$$d\vec{E}_x = \frac{2\pi\sigma \ kxrdr}{(x^2 + r^2)^{\frac{3}{2}}}\hat{x}$$

Integrating both sides

$$\vec{E}_{x} = \int_{0}^{R} 2\pi\sigma \, kx \, \frac{r dr}{(x^{2} + r^{2})^{\frac{3}{2}}} \hat{x}$$

We change the variable and put $u = x^2 + r^2$. Then du = 2r. dr, i.e., r. dr = du/2. Also, when $r = 0, u = x^2$ and when $r = R, u = x^2 + R^2$

$$\vec{E}_x = \pi x \sigma k \int_{x^2}^{x^2 + R^2} \frac{du}{u^{\frac{3}{2}}} \hat{x}$$



$$\vec{E}_x = \pi x k \sigma \left[\frac{-2}{u^{\frac{1}{2}}}\right]_{x^2}^{x^2 + R^2} \hat{x}$$
$$\vec{E}_x = 2\pi x k \sigma \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}}\right] \hat{x}$$
$$\vec{E}_x = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{1}{\sqrt{1 + \frac{R^2}{x^2}}}\right] \hat{x}$$

A graph of \vec{E}_x against x has the form, When R >> x, i.e., the plate is very large $\vec{E}_x \to \frac{\sigma}{2\varepsilon_0} \hat{x}$



2.4 Electric field lines

An electric field line is the path taken by a small positive charge placed in the field

2.4.2 Properties of electric field lines

- 1. The electric field line starts from the positive charge and ends from negative charge
- 2. Field lines never intersect each other
- 3. They are in a state of tension which causes them to shorten
- 4. They are perpendicular to the surface charge.
- 5. The field is strong when the lines are close together, and it is weak when the field lines move apart from each other.
- 6. They repel one another side ways



- 7. The number of field lines is directly proportional to the magnitude of the charge.
- 8. The line curves are continuous in a charge-free region
- 9. If the charge is single, then they start or end at infinity.

2.5 The sketches below represent electric field patterns of some common

charge distribution





2.5 Electric flux

This is the product of electric field strength at any point and area normal to the field i.e.

$$\phi_E = \vec{E}A$$



Units: Nm^2/C

However, if the E-field lines lie at an angle θ with respect to the normal line of area A (see Figure below), the electric flux is given by the more general formula:



$\phi_E = \vec{E}.A = EAcos\theta$

Where \vec{A} has magnitude A (the total cross-sectional or surface area) and direction given by the normal line of the area.

The angle θ is the angle between that normal line and the E-field direction

Note: If $\theta = 0$ Then $\phi_E = \vec{E} \cdot A$ If $\theta = 90^0$ Then $\phi_E = 0$

Example	1	
A uniform electric field of $\vec{E} = a\hat{\imath} + b\hat{\jmath}$ intersects a surface of area \vec{A} . Find the flux through		
this area i	f the surface lies in the	
(i)	yz plane,	
(ii)	xz plane,	
(iii)	xy plane.	



Solution



(i) Surface in the yz plane



(ii) Surface in the xz plane

$$\vec{A} = A\hat{j}$$

$$\hat{k}$$

$$\hat{j}$$

$$\hat{i}$$

$$\phi = \vec{E} \cdot \vec{A}$$

 $\phi = (a\hat{\imath} + b\hat{\jmath}).(A\hat{\jmath})$



 $\phi = bA$

(iii) Surface in the xz plane



Example

Calculate the electric flux through a sphere that has a radius of 100cm and carries a charge of $2.0\mu C$ at its centre

Solution

The magnitude of Eclectic field is $\vec{E} = k \frac{q}{r^2}$

$$\vec{E} = 9.0x10^9 x \frac{2.0x10^{-6}}{(100x10^{-2})^2}$$
$$\vec{E} = 18000N/C$$
$$\vec{A} = 4\pi r^2$$
$$\vec{A} = 4x3.14x(100x10^{-2})^2$$
$$\vec{A} = 12.6m^2$$

The electric flux is

The area of the sphere is

 $\phi = \vec{E}.\vec{A}$

 $\phi = 12.6x18000$



$\phi = 226080 Nm^2/C$

2.7 Gauss's law

If we use a sphere for our enclosing volume, the sphere has a surface area of $A = 4\pi r^2$ If we place charge q at the center of this sphere, then the field lines will always be perpendicular to the surface of the sphere since they point radially outward. Using Equation of electric field intensity we can write the electric flux as

$$\phi = \vec{E} \cdot \vec{A}$$

$$\phi = \frac{q}{4\varepsilon_0 \pi r^2} x 4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_0}$$

The equation is called Gauss's law of electrostatics

In other words, this law states: The electric flux through any closed surface is equal to the net charge q inside the surface divided by the permittivity of free space ε_0 .

2.8 Examples of applications of Gauss's law

2.8. 1 Electric field intensity outside a charged conducting sphere of radius R in air





Consider the gaussian surface as a spherical surface S of radius r concentric with a charged sphere. By symmetry the electric field intensity is everywhere normal to s

By gauss's law

$$\oint_{S} \vec{E}\hat{n}dS = E(4\pi r^{2}) = \frac{Q}{\varepsilon_{0}}$$
$$\vec{E} = \frac{Q}{4\pi\varepsilon_{0}r^{2}}\hat{r}, \text{ for } r > R$$

Where \hat{r} is the unit vector a long the radial direction

2.8.3 Electric field intensity inside a charged conducting sphere

Take a charged conducting sphere. There is no net charge inside the sphere all the net charge resides on the surface.



Consider the gaussian surface as the spherical surface S of radius R > r having Centre O, the Centre of the conducting sphere

By gauss's law

$$\oint_{S} \vec{E}\hat{n}dS = 0$$

(since there is no net charge inside S)

 $\vec{E}(4\pi r^2) = 0$



$$\vec{E} = 0$$
, for $R > r$

The electric field intensity inside the conducting sphere is zero

A graph of E against r from the Centre of the sphere



Note

- (i) Outside the charged sphere: Since $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$ then $\vec{E} \propto \frac{1}{r^2}$
- (ii) Inside the Sphere: No charge resides on the inside of a hollow conductor therefore E = 0

2.8.4 Electric field intensity just outside a uniformly charged conductor

Take the gaussian surface, an imaginary pull-box of cross-sectional area A under electrostatic conditions, E must be normal to the conductor





By symmetry there is no electric flux through the curved surface of the pail box, E is normal to the end cap of the pull box

By gauss's law

$$\int_{S} \vec{E} \hat{n} dS = \frac{\sigma A}{\varepsilon_0}$$
$$E(A) = \frac{\sigma A}{\varepsilon_0}$$
$$E = \frac{\sigma}{\varepsilon_0} \hat{n}$$

Where \hat{n} is the unit vector normal to the plane conductor.

2.8.5 Electric field intensity inside and outside a spherical distribution of charge density ρCm^{-3} and radius R

Consider points inside the distribution, and take for the Gaussian surface, a spherical surface S of the radius r < R





By gauss's law

$$\oint_{S} \vec{E}\hat{n}dS = \frac{Q}{\varepsilon_0}$$

Where $Q = \int_{\tau} \rho d\tau = \rho \left(\frac{4\pi r^3}{3}\right)$ is the total charge enclosed by S

Because of spherical symmetry, the electric field intensity E is the same at all points on S, Hence

$$\oint_{S} \vec{E}\hat{n}dS = E(4\pi r^{2})$$
$$E(4\pi r^{2}) = \rho\left(\frac{4\pi r^{3}}{3\varepsilon_{0}}\right)$$
$$E = \frac{\rho r}{3\varepsilon_{0}}\hat{r} \text{ for } r < R$$







By gauss's law

$$\oint_{S} \vec{E} \hat{n} dS = \frac{Q}{\varepsilon_0}$$

Where $Q = \int_{\tau} \rho d\tau = \rho \left(\frac{4\pi R^3}{3}\right)$ is the total charge enclosed by S Because of spherical symmetry, the electric field intensity E is the same

Because of spherical symmetry, the electric field intensity E is the same at all points on S, Hence

$$\oint_{S} \vec{E}\hat{n}dS = E(4\pi r^{2})$$
$$E(4\pi r^{2}) = \rho\left(\frac{4\pi R^{3}}{3\varepsilon_{0}}\right)$$
$$E = \frac{\rho R^{3}}{3\varepsilon_{0}r^{2}}\hat{r} \text{ for } r > R$$

A graph of E against r from the Centre of the sphere has the form





2.8.6 Electric field intensity at a point perpendicular distance r from a line of



The electric field intensity must be radially outwards. Hence the flux through the end caps of the gaussian cylinder is zero

The flux through the curved surface is equal to $(2\pi rL)E$

$$\oint_{S} \vec{E}\hat{n}dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

Since the total charge enclosed by S is equal λL Hence $E = \frac{\lambda}{2\pi\varepsilon_0}$

2.8.7 Electric field intensity inside and outside a charged conducting shell

(i) Inside the shell

Take as gaussian surface a spherical surface S, of radius r having the same center as the spherical surface

charge of λCm^{-1}





By gauss's law
$$\oint_{S} \vec{E} \hat{n} dS = 0$$

(since S does not enclose any charge) $\vec{E}(4\pi r^2) = 0$ $\vec{E} = 0$, for R > r

(ii) Outside the shell





By gauss's law

$$\oint_{S} \vec{E}\hat{n}dS = E(4\pi r^{2}) = \frac{Q}{\varepsilon_{0}}$$
$$\vec{E} = \frac{Q}{4\pi\varepsilon_{0}r^{2}}\hat{r}, \text{ for } r > R$$

A sketch graph of E against r has the form shown



Self-Review Question (SRQs) for study unit 2

Now that you have completed this unit, you can measure how well you achieved its learning outcomes by answering the following questions. You can check your answers with the Notes on Self-Review Questions at the end of this study unit

- 1. Define electric field intensity
- 2. A charge $q_1 = 9.00\mu C$ is located at the origin, and a second charge $q_2 = 8.00\mu C$ is located on the x-axis, 0.40m from the origin. Find the electric field at the point P, which has coordinates (0,0.50) m.
- 3. A particle carrying charge of $+3.0\mu C$ is placed in an electric field of intensity $1.0x10^3 N/C$. find the electric force on the particle
- 4. A particle of charge $q_1 = 9.60 \mu C$ is stationary under the action of uniform electric and gravitational forces. The mass of the particle is 4.0g. How long is the electric field intensity?



- 5. Two-point charges of $+9.00\mu C$ and $-6.00\mu C$ are separated by a distance of 0.20m. at what point along the line the charges is the electric field intensity is zero
- 6. Find electric field intensity at point C



The electric field intensity at a point from a point charge of 4.8nC is $3.6x10^3 N/C$. How far is the point charge from this point

- 7. A rod of length l with uniform charge per unit length λ is placed at a distance d from origin along the x-axis. A similar rod is placed at the same distance along Y-axis. Determine the magnitude of net electric field intensity at the origin.
- 8. Sketch electric field lines in the vicinity of two negative point charges placed near each other
- 9. A disc of radius 5.0cm is uniformly charged with a total charge of $12.0\mu C$. find the electric field intensity at a point on the axis of the ring and the distance of 8.0cm from the disc
- 10. A ring of radius of 4.0cm carries a charge of $10.0\mu C$ uniformly distributed on it. Find the electric field intensity at a point on the axis of the ring and 10.0cm from the centre of the ring.
- 11. A plane area of $0.40m^2$ makes an angle of 30^0 with a uniform electric field of intensity $2.0x10^3N/C$. Find electric flux through the area
- 12. Show that the electric field intensity just outside a charged, non-conducting infinite sheet is $\frac{\sigma}{2\varepsilon_0}$, where σ is the surface charge density
- 13. A spherical charge distribution is given by

$$\rho(r) = \begin{cases} \rho_0 \left(1 - \frac{r^2}{a^2} \right), r < a \\ 0, r > a \end{cases}$$

Where a is radius and r is the distance from the center of the distribution



- (i) Find the total charge Q
- (ii) Determine the electric field intensity both inside and outside the distribution



Summary

- > The electric field at a point in space is defined as the electric force exerted on a test charge placed at that point. $\vec{E} = \frac{\vec{F}_e}{q_0}$ The SI units of \vec{E} are NC^{-1} .
- > The electric field of a point charge q is given by $\vec{E} = k \frac{q}{r^2} \hat{r}$ Where \hat{r} is a unit vector pointing from the point charge q to the location at which the electric field is being calculated.
- The electric field due to a distribution of charges, according to the superposition principle, is the vector sum of the fields of the individual charges making up the distribution
- > Gauss's law and its application is also discussed on Unit 1



References

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